

Radius of Convergence (Section 10.4)

* Convergence Theorem for Power Series: only 3 possibilities for $\sum_{n=0}^{\infty} c_n (x-a)^n$ with respect to convergence:

1) There is a positive number, R , such that $|x-a| > R$ diverges \neq $|x-a| < R$ converges.

* The series may or may not converge at the end-pts: $x = a - R \neq x = a + R$.

2) If $R = \infty$, the series converges for every x .

3) If $R = 0$, the series converges @ $x = a \neq$ diverges elsewhere.

* Radius of Convergence (R): set of values where the series converge.

* Interval of Convergence: $|x-a| < R$ or $a-R < x < a+R$
(can be open, closed, or half-open) ← Radius of Convergence
↑ series is centered @ "a"

* n^{th} Term Test for Divergence:

$\sum_{n=1}^{\infty} a_n$ diverges if $\lim_{n \rightarrow \infty} a_n$ fails to exist OR is different from zero.

****** This test will not tell you if the series converges. If $\lim_{n \rightarrow \infty} a_n = 0$, it may or may not converge! → move on!

ex: $\sum_{n=1}^{\infty} \frac{4n^2 + n^3}{7 + 3n^3}$

* Direct Comparison Test: Let $\sum a_n$ be a series with no negative terms.

a) $\sum a_n$ converges if $\sum c_n$ converges and $a_n \leq c_n$

b) $\sum a_n$ diverges if $\sum d_n$ diverges and $a_n \geq d_n$

ex: $\sum_{n=0}^{\infty} \frac{x^{2n}}{(n!)^2}$

* Ratio Test: Let $\sum a_n$ be a series w/ all positive terms.

If $\lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = L$, then

a) the series converges if $L < 1$

b) the series diverges if $L > 1$

c) the test is inconclusive if $L = 1$.
 \rightarrow move on!

* Exp #1 on pg 512: $\sum_{n=1}^{\infty} \frac{1}{n} \neq \sum_{n=1}^{\infty} \frac{1}{n^2}$

1) Ratio test:

2) $\int_1^{\infty} \frac{1}{x} dx =$

$$\int_1^{\infty} \frac{1}{x^2} dx$$

Exp #1 can't

3) $\sum \frac{1}{n}$ diverges but $\sum \frac{1}{n^2}$ converges. why?

4) Both ratio tests $\Rightarrow L=1$ and one series converged & the other diverged, this is why $L=1$ is inconclusive!

*Exp #2 pg 513:
$$x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots + \frac{(-1)^{n-1} x^n}{n} = \sum_{n=1}^{\infty} \frac{(-1)^{n-1} x^n}{n}$$

1) **Ratio test:** *ignore the alternating sign (just pos. terms)

2) Left end-pt: $x = -1$

3) Right end-pt: $x = 1$

ex: #19 $\sum_{n=0}^{\infty} (-2)^n (n+1)(x+1)^n$