Section 10.5 (Day 2)

* **Absolute Convergence**: If $\sum |a_n|$ converges, then $\sum a_n$ converges absolutely.

  ↓

  Absolute convergence implies convergence.

* In other words... If the series converges w/o alternating, then the series converges absolutely.

* A series converges conditionally if it needs the alternating part to converge.

ex: #23 $\displaystyle\sum_{n=1}^{\infty} (-1)^{n+1} \cdot \frac{1+n}{n^2}$

Absolute Convergence: $\displaystyle\sum_{n=1}^{\infty} \frac{1+n}{n^2} = \sum_{n=1}^{\infty} \frac{1}{n^2} + \frac{1}{n}$ → Direct Comparison

$\frac{1}{n^2} + \frac{1}{n} > \frac{1}{n}$ ∴ since $\frac{1}{n}$ div. $\frac{1}{n^2} + \frac{1}{n}$ also diverge

Now check conditional convergence:

Alt. Series test: $u_n = \frac{1+n}{n^2}$

positive? ✓

decreasing? ✓

$\lim_{n\to\infty} \frac{1+n}{n^2} = 0$? ✓

∴ the series converges conditionally.

ex: $\displaystyle\sum_{n=0}^{\infty} \frac{(\sin x)^n}{n!}$

Absoute Convergence: $\displaystyle\sum_{n=0}^{\infty} \frac{|\sin x|^n}{n!}$ → $|\sin x|$ is bounded between 0 & 1

$$0 \le \sum_{n=0}^{\infty} \frac{|\sin x|^n}{n!} \le \underbrace{\sum_{n=0}^{\infty} \frac{1^n}{n!}}_{\text{This converges to } e^1}$$

∴ $\displaystyle\sum_{n=0}^{\infty} \frac{|\sin x|^n}{n!}$ converges so...

$\displaystyle\sum_{n=0}^{\infty} \frac{(\sin x)^n}{n!}$ converges absolutely!

* FLOW CHART: ABSOLUTE -VS- CONDITIONAL

① Drop the alternating part / absolute value the function.

② Use a test to prove converge/diverge

CONVERGE → $\boxed{\text{ABSOLUTELY CONVERGES}}$

DIVERGE → Check for Conditional

① Go back to the original series

② Do Alternating Series Test.

u_n: ① positive?

② decreasing?

③ $\lim_{n\to\infty} u_n = 0$?

③ If yes → converge → $\boxed{\text{CONDITIONAL CONVERGENT}}$

If no → $\boxed{\text{DIVERGES}}$

#39 $\sum_{n=0}^{\infty} \frac{(x-2)^n}{10^n} = \sum_{n=0}^{\infty} \left(\frac{x-2}{10}\right)^n$ ← geometric!

$$\left|\frac{x-2}{10}\right| < 1$$

$$-1 < \frac{x-2}{10} < 1$$

$$-10 < x-2 < 10$$
$$+2 \quad\quad +2 \quad +2$$
$$-8 < x < 12$$

Check End-Points!

@ $x=-8$: $\sum_{n=0}^{\infty} \left(\frac{-8-2}{10}\right)^n = \sum_{n=0} (-1)^n$ → diverge!

@ $x=12$: $\sum_{n=0}^{\infty} \left(\frac{12-2}{10}\right)^n = \sum_{n=0}^{\infty} (1)^n$ → diverge!

Interval of Convergence: $(-8, 12)$

* For the book: Absolute Convergence → both endpoints converge or both diverge
Conditional Convergence → one endpoint converges & one diverges.

Series Test Summary

* n^{th} Term Test: $\lim_{n\to\infty} a_n \to \neq 0$

* Ratio Test: $\lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| \to |L| < 1$ ** Don't forget to check the end-points!

* Direct Comparison: show a relationship between unknown & known series: $\frac{3}{n^2+3} < \frac{3}{n^2} \to$ converges by p-Series

* Integral Test: ① Continuous? ② Decreasing? ③ Positive? } Take the integral

* p-Series: $n>1 \to$ converge if $n \leq 1 \to$ diverge

* LCT: end behavior $\to \frac{\text{BIG DOG}}{\text{BIG DOG}} \to$ reduce it: does it conv/diverge?

$\lim_{n\to\infty} \frac{a_n}{b_n}$ known

* Alternating Series Test: ① Positive? ② Decreasing? ③ $\lim_{n\to\infty} u_n = 0$? If yes $\to$ converge!

* Geometric / Power Series: Sum $= \frac{a_0}{1-r}$ $|r|<1$

Geometric $\downarrow$ $\sum r^n$

Power Series $\downarrow$ $\sum a_0 r^n$