Testing Convergence @ Endpoints (Section 10.5)

* Integral Test: If a function is continuous, positive & decreasing, then you can take the integral of it.
  ** Either both the integral & the series converge or both diverge.

ex: $\sum_{n=1}^{\infty} \frac{1}{n\sqrt{n}}$ Converge or Diverge?

Is it continuous? ✓
positive? ✓
decreasing? ✓

$$\int_1^{\infty} \frac{1}{n\sqrt{n}}\,dn = \lim_{b\to\infty} \int_1^b x^{-3/2}\,dx$$

$= n^{3/2}$

$$= \lim_{b\to\infty} -2x^{-1/2} \Big|_1^b$$

$$= \lim_{b\to\infty} \left(\frac{-2}{\sqrt{b}} + \frac{+2}{\sqrt{1}}\right) = 0+2 = 2$$

$\therefore$ the series converges.

** p-Series Test: $\sum_{n=1}^{\infty} \frac{1}{n^p}$ if $p > 1 \rightarrow$ converges
$p \leq 1 \rightarrow$ diverges

ex. $\sum_{n=1}^{\infty} \frac{1}{n\sqrt{n}} = \sum_{n=1}^{\infty} \frac{1}{n^{3/2}}$ p-Series: $\frac{3}{2} > 1 \therefore$ converge!

* Exploration #1 (pg. 518)

1. $p > 1 \rightarrow$ Integral Test: $\int_1^{\infty} \frac{1}{x^p}\,dx = \lim_{b\to\infty} \int_1^b x^{-p}\,dx$

$\downarrow$

$p - 1 > 0$

$-(p-1)$

$$= \lim_{b\to\infty} \frac{x^{-p+1}}{-p+1} \Big|_1^b$$

$$= \frac{1}{-p+1} \lim_{b\to\infty} \frac{1}{x^{p-1}} \Big|_1^b$$

$$= \frac{1}{-p+1} \lim_{b\to\infty} \left(\frac{1}{b^{p-1}} - \frac{1}{1^{p-1}}\right)$$

*since $p-1>0$, the "b" stays in the denom.

$$\frac{1}{-p+1}(0-1) = \frac{-1}{-p+1} = \frac{1}{p-1}$$

$\therefore$ converges!

#2 $p<1$ $\downarrow$ $p-1<0$

Integral Test: *all would be the same until:

$$\frac{1}{-p+1}\lim_{b\to\infty}\left(\frac{1}{b^{p-1}}-\frac{1}{1}\right)$$

*if $p-1<0$, the "b" would move into the numerator and the limit would go to ∞.

$\therefore$ it must diverge.

#3 $p=1$ Integral Test: $\int_1^\infty \frac{1}{x^1}\,dx$

$$=\lim_{b\to\infty}\int_1^b x^{-1}\,dx=\lim_{b\to\infty}\ln|x|\Big|_1^b$$

$$=\lim_{b\to\infty}\ln b-\cancel{\ln 1}^{\,0}=\ln\infty=\infty$$

$\therefore$ diverge.

* Limit Comparison Test (LCT): $\sum a_n$ \& $\sum b_n$ → known are both positive series/sums then:

↳ think end behavior!

1- $\lim_{n\to\infty}\frac{a_n}{b_n}=c$ and $0<c<\infty$ → both converge or both diverge

2- $\lim_{n\to\infty}\frac{a_n}{b_n}=0$ and $\sum b_n$ converges, then $\sum a_n$ also converges.

3- $\lim_{n\to\infty}\frac{a_n}{b_n}=\infty$ and $\sum b_n$ diverges, then $\sum a_n$ also diverges.

ex: $\sum_{n=2}^{\infty}\frac{3n+2}{n^3-2n}$ → LCT: end behavior $=\frac{3}{n^2}\to\frac{1}{n^2}$

$$\lim_{n\to\infty}\left(\frac{\frac{3n+2}{n^3-2n}}{\frac{1}{n^2}}\right)=\lim_{n\to\infty}\left[\left(\frac{3n+2}{n^3-2n}\right)\left(\frac{n^2}{1}\right)\right]=\lim_{n\to\infty}\left(\frac{3n^3+2n^2}{n^3-2n}\right)=\frac{3\cancel{n^3}}{\cancel{n^3}}=3$$

$\therefore$ since $\frac{1}{n^2}$ converges, then the series $\sum_{n=2}^{\infty}\frac{3n+2}{n^3-2n}$ must also converge!

** Alternating Series Test: $\sum_{n=1}^{\infty}(-1)^{n+1}\cdot u_n = u_1 - u_2 + u_3 - u_n + \dots$

Converges if ALL 3 Conditions are met:

① each u_n is positive

② $u_n \geq u_{n+1}$ (each term decreases)

③ $\lim_{n\to\infty} u_n = 0$

* Conditional Convergence: when one end-pt converges & the other diverges

* Absolute convergence: both end-pts converge or both diverge.

** Alternating Harmonic Series: $\boxed{\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n}}$ → converges

N Day 1

#1 $\int_1^{\infty} 1/x^{4/3}\,dx$ → p-Series: $4/3 > 1$ ∴ converge

#2 $\int_1^{\infty}\frac{x^2}{x^3+1}$ → LCT: end beh = $\frac{1}{x}$ ∴ diverge

#8 $f(x)=\frac{3+x^2}{3-x^2}$ where is it both pos. & decr.

$\frac{3+x^2}{3-x^2} > 0$ when $3-x^2>0$

$3 > x^2$

$\sqrt{3} > x$ or $-\sqrt{3} < x$

$-\sqrt{3} < x < \sqrt{3}$