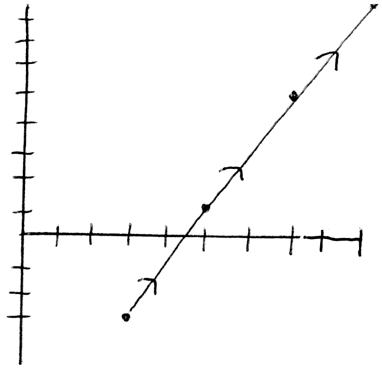
Section 11.1: 1–39 odd

1. $x = 2t+3$
   $y = 4t-3$
   $[0,3]$

t	0	1	2	3
x	3	5	7	9
y	-3	1	5	9



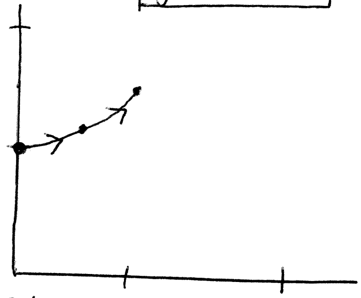
$\boxed{\text{Yes}}$ this graph is a function.

$x = 2t+3 \to 2t = -3+x \to t = \frac{-3}{2} + \frac{1}{2}x$

$y = 4t-3 \to y = 4\left(\frac{-3}{2} + \frac{1}{2}x\right) - 3 \to y = -6 + 2x - 3 \to \boxed{y = 2x-9}$

3. $x = \tan t$
   $y = \sec t$
   $[0, \pi/4]$

t	0	$\pi/6$	$\pi/4$
x	0	$\sqrt{3}/3$	1
y	1	$2\sqrt{3}/3$	$\sqrt{2}$



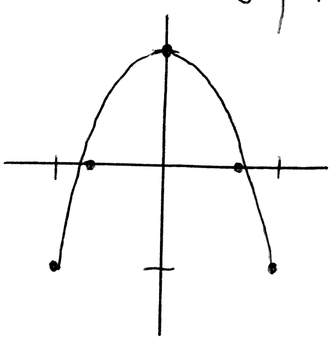
$\boxed{\text{Yes}}$ this graph is a function.

$\sin^2 t + \cos^2 t = 1 \to \tan^2 t + 1 = \sec^2 t \to \sec^2 t - \tan^2 t = 1 \to y^2 - x^2 = 1$

$y^2 = 1 + x^2 \to \boxed{y = \sqrt{1+x^2}}$

5. $x = \sin t$
   $y = \cos 2t$
   $[0, 2\pi]$

t	0	$\pi/4$	$\pi/2$	$3\pi/4$	π	$5\pi/4$	$3\pi/2$	$7\pi/4$
x	0	$\sqrt{2}/2$	1	$\sqrt{2}/2$	0	$-\sqrt{2}/2$	-1	$-\sqrt{2}/2$
y	1	0	-1	0	1	0	-1	0



$\boxed{\text{Yes}}$ this graph is a function

$x = \sin t$
$y = \cos 2t$
$y = 1 - 2\sin^2 t$ (trig identity)
$\boxed{y = 1 - 2x^2}$

7. $x = 4\sin t$, $y = 2\cos t$

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{-2\sin t}{4\cos t} = \boxed{\frac{-1}{2}\tan t}$$

$$\frac{d^2y}{dx^2} = \frac{(dy/dx)'}{dx/dt} = \frac{-1/2\sec^2 t}{4\cos t} = \frac{-1/8\sec^2 t}{\frac{1}{\sec t}} = \boxed{\frac{-1}{8}\sec^3 t}$$

9. $x = -\sqrt{t+1}\,,\ y = \sqrt{3t}$ $\qquad 3^2 = 9$

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{\cancel{\tfrac{1}{2}}(3t)^{-1/2}\cdot 3}{-\cancel{\tfrac{1}{2}}(t+1)^{-1/2}} = \frac{-3\sqrt{t+1}}{\sqrt{3t}} = \frac{-\sqrt{9t+9}}{\sqrt{3t}} = -\sqrt{\frac{9t+9}{3t}} = \boxed{-\sqrt{3+\frac{3}{t}}}$$

$$\frac{d^2y}{dx^2} = \frac{(dy/dx)'}{dx/dt} = \frac{\cancel{+\tfrac{1}{2}}(3+3/t)^{-1/2}\cdot\frac{-3}{t^2}}{\cancel{+\tfrac{1}{2}}(t+1)^{-1/2}} = \frac{-3\sqrt{t+1}}{t^2\sqrt{3+3/t}} = \boxed{\frac{-3}{t^2}\sqrt{\frac{t+1}{3+3/t}}}$$

11. $x = t^2 - 3t,\ y = t^3$

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \boxed{\frac{3t^2}{2t-3}}$$

$$\frac{d^2y}{dx^2} = \frac{(dy/dx)'}{dx/dt} = \frac{\frac{(2t-3)6t - 3t^2(2)}{(2t-3)^2}}{2t-3} = \frac{12t^2-18t-6t^2}{(2t-3)^3} = \boxed{\frac{6t^2-18t}{(2t-3)^3}}$$

13. $x = \tan t,\ y = \sec t$

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{\cancel{\sec t}\cdot\tan t}{\sec^{\cancel{2}} t} = \frac{\tan t}{\sec t} = \frac{\sin t}{\cancel{\cos t}}\cdot\frac{\cancel{\cos t}}{1} = \boxed{\sin t}$$

$$\frac{d^2y}{dx^2} = \frac{(dy/dx)'}{dx/dt} = \frac{\cos t}{\sec^2 t} = \frac{\cos t}{1}\cdot\frac{\cos^2 t}{1} = \boxed{\cos^3 t}$$

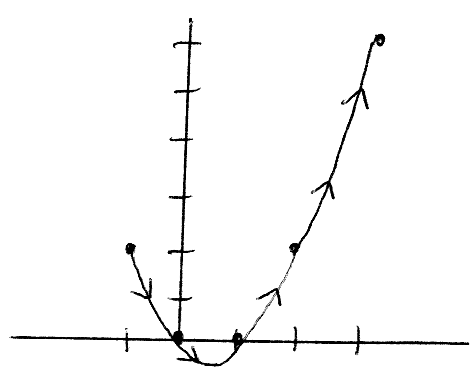
15. $x = \ln(2t),\ y = \ln(3t)^4 = 4\ln(3t)$

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{4\cdot\frac{1}{3t}\cdot 3}{\frac{1}{2t}\cdot 2} = \frac{12/3t}{2/2t} = \frac{4/t}{1/t} = \frac{4}{\cancel{t}}\cdot\frac{\cancel{t}}{1} = \boxed{4}$$

$$\frac{d^2y}{dx^2} = \frac{(dy/dx)'}{dx/dt} = \frac{0}{1/t} = \boxed{0}$$

17. $x = t+1$
$y = t^2+t$
$[-2, 2]$

t	-2	-1	0	1	2
x	-1	0	1	2	3
y	2	0	0	2	6

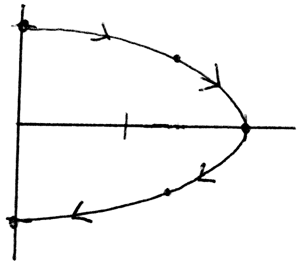


Lowest: Minimize $y \to \frac{dy}{dt} = 2t+1 = 0 \to t = -1/2$

$x(-1/2) = -1/2 + 1 = 1/2$
$y(-1/2) = 1/4 - 1/2 = -1/4$ $\Big\rangle$ $\boxed{(1/2, -1/4)}$

19. $x = 2\sin t, \quad y = \cos t, \quad 0 \le t \le \pi$

t	0	$\pi/4$	$\pi/2$	$3\pi/4$	π
x	0	$\sqrt{2}$	2	$\sqrt{2}$	0
y	1	$\sqrt{2}/2$	0	$-\sqrt{2}/2$	-1

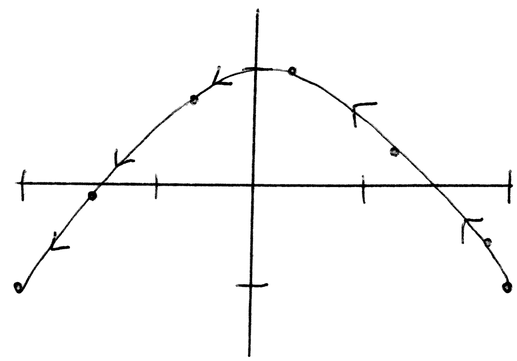


Rightmost: maximize $x \to \frac{dx}{dt} = 2\cos t = 0 \to \cos t = 0 \to t = \pi/2$

$x(\pi/2) = 2\sin\pi/2 = 2\cdot 1 = 2$
$y(\pi/2) = \cos\pi/2 = 0$ $\to \boxed{(2,0)}$

21. $x = 2\sin t, \quad y = \cos(2t), \quad 1.5 \le t \le 4.5$

t	1.5	2	2.5	3	3.5	4	4.5
x	2.0	1.8	1.2	0.3	-0.7	-1.5	-2.0
y	-1.0	-0.7	0.3	1.0	0.8	-0.1	-0.9



Highest: maximize $y \to \frac{dy}{dt} = -2\sin 2t = 0 \to \sin 2t = 0 \to 2t = \pi \to t = \pi/2$
$2t = 2\pi \to t = \pi$

$x(\pi/2) = 2\sin\pi/2 = 2\cdot 1 = 2$
$y(\pi/2) = \cos\pi = -1$ $\to (2,-1) \to$ This is the min y, not the max

$x(\pi) = 2\sin\pi = 2\cdot 0 = 0$
$y(\pi) = \cos 2\pi = 1$ $\to \boxed{(0,1)} \to$ Max y

23. $x = 2+\cos t, \quad y = -1+\sin t$

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{\cos t}{-\sin t}$$

a) Horizontal when $\cos t = 0 \to t = \pi/2,\ t = 3\pi/2$

$x(\pi/2) = 2+0 = 2, \quad y(\pi/2) = -1+1 = 0 \to \boxed{(2,0)}$
$x(3\pi/2) = 2+0 = 2, \quad y(3\pi/2) = -1-1 = -2 \to \boxed{(2,-2)}$

b) Vertical when $-\sin t = 0 \to t = 0,\ t = \pi$

$x(0) = 2+1 = 3, \quad y(0) = -1+0 = -1 \to \boxed{(3,-1)}$
$x(\pi) = 2-1 = 1, \quad y(\pi) = -1+0 = -1 \to \boxed{(1,-1)}$

25. $x = 2-t,\ y = t^3-4t$

$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{3t^2-4}{-1} = -3t^2+4$

a) Horizontal when $-3t^2+4=0 \rightarrow 3t^2=4 \rightarrow t^2 = 4/3 \rightarrow t = \pm\sqrt{4/3}$

$x\left(\sqrt{\tfrac{4}{3}}\right) = 2-\sqrt{\tfrac{4}{3}} = 0.845$

$y\left(\sqrt{\tfrac{4}{3}}\right) = \sqrt{\tfrac{4}{3}}^3 - 4\sqrt{\tfrac{4}{3}} = -3.079$

$\boxed{(0.845, -3.079)}$

$x\left(-\sqrt{\tfrac{4}{3}}\right) = 2+\sqrt{\tfrac{4}{3}} = 3.155$

$y\left(-\sqrt{\tfrac{4}{3}}\right) = \left(-\sqrt{\tfrac{4}{3}}\right)^3 + 4\sqrt{\tfrac{4}{3}} = 3.079$

$\boxed{(3.155, 3.079)}$

b) Vertical when denominator $= 0 \rightarrow$ $\boxed{\text{never}}$

27. $x = \cos t,\ y = \sin t,\ 0 \le t \le 2\pi$

$$L = \int_0^{2\pi} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$

$\frac{dx}{dt} = -\sin t \rightarrow \left(\frac{dx}{dt}\right)^2 = \sin^2 t,\quad \frac{dy}{dt} = \cos t \rightarrow \left(\frac{dy}{dt}\right)^2 = \cos^2 t$

$$L = \int_0^{2\pi} \sqrt{\sin^2 t + \cos^2 t}\, dt = \int_0^{2\pi} \sqrt{1}\, dt = \int_0^{2\pi} 1\, dt = t\Big|_0^{2\pi} = 2\pi - 0 = \boxed{2\pi}$$

29. $x = 8\cos t + 8t\cdot\sin t,\ y = 8\sin t - 8t\cdot\cos t,\ 0 \le t \le \pi/2$

$\frac{dx}{dt} = \cancel{-8\sin t} + 8t\cos t + \cancel{8\sin t} = 8t\cos t \rightarrow \left(\frac{dx}{dt}\right)^2 = 64t^2\cos^2 t$

$\frac{dy}{dt} = \cancel{8\cos t} + 8t\sin t - \cancel{8\cos t} = 8t\sin t \rightarrow \left(\frac{dy}{dt}\right)^2 = 64t^2\sin^2 t$

$$L = \int_0^{\pi/2} \sqrt{64t^2\underbrace{(\cos^2 t + \sin^2 t)}_{1}}\, dt = \int_0^{\pi/2} \sqrt{64t^2}\, dt = \int_0^{\pi/2} 8t\, dt = 4t^2\Big|_0^{\pi/2}$$

$4\left(\frac{\pi}{2}\right)^2 - \cancel{4\cdot 0^2} = \cancel{4}\cdot\frac{\pi^2}{\cancel{4}} = \boxed{\pi^2}$

31. $x=\frac{1}{3}(2t+3)^{3/2}; \quad y=t+\frac{1}{2}t^2, \quad 0\le t\le 3$

$$\frac{dx}{dt}=\cancel{\frac{1}{2}}(2t+3)^{1/2}\cdot\cancel{2}=\sqrt{2t+3}\rightarrow\left(\frac{dx}{dt}\right)^2=\sqrt{2t+3}^{\,2}=2t+3$$

$$\frac{dy}{dt}=1+t\rightarrow\left(\frac{dy}{dt}\right)^2=t^2+2t+1$$

$$L=\int_0^3\sqrt{t^2+4t+4}\,dt=\int_0^3\sqrt{(t+2)^2}\,dt=\int_0^3(t+2)dt=\left(\frac{1}{2}t^2+2t\right)\Big|_0^3$$

$$L=\left(\frac{9}{2}+6\right)-(0+0)=\frac{9}{2}+\frac{12}{2}=\boxed{\frac{21}{2}}$$

33. $x=\frac{1}{3}t^3, \quad y=\frac{1}{2}t^2, \quad 0\le t\le 1$

$$\frac{dx}{dt}=t^2\rightarrow\left(\frac{dx}{dt}\right)^2=t^4$$

$$\frac{dy}{dt}=t\rightarrow\left(\frac{dy}{dt}\right)^2=t^2$$

$$L=\int_0^1\sqrt{t^4+t^2}\,dt=\int_0^1\sqrt{t^2(t^2+1)}\,dt=\int_0^1 t\sqrt{t^2+1}\,dt\rightarrow \begin{aligned}u&=t^2+1\\ du&=2t\,dt\\ dt&=\frac{du}{2t}\end{aligned}$$

$$L=\int\cancel{t}\sqrt{u}\cdot\frac{du}{2\cancel{t}}=\int\frac{1}{2}u^{1/2}du=\frac{1}{\cancel{2}}\cdot\frac{\cancel{2}}{3}u^{3/2}=\frac{1}{3}u^{3/2}=\frac{1}{3}(t^2+1)^{3/2}\Big|_0^1$$

$$L=\frac{1}{3}\cdot2^{3/2}-\frac{1}{3}=\boxed{\frac{2^{3/2}-1}{3}\approx0.609}$$

35. a) $x=\cos2t, \quad y=\sin2t, \quad 0\le t\le\pi/2$

$$\frac{dx}{dt}=-2\sin2t\rightarrow\left(\frac{dx}{dt}\right)^2=4\sin^2 2t$$

$$\frac{dy}{dt}=2\cos2t\rightarrow\left(\frac{dy}{dt}\right)^2=4\cos^2 2t$$

$$L=\int_0^{\pi/2}\sqrt{4(\underbrace{\sin^2 2t+\cos^2 2t}_{1})}\,dt=\int_0^{\pi/2}\sqrt{4}\,dt=\int_0^{\pi/2}2\,dt=2t\Big|_0^{\pi/2}=\cancel{2}\frac{\pi}{\cancel{2}}=\boxed{\pi}$$

35. b) $x = \sin\pi t,\ y = \cos\pi t,\ -1/2 \le t \le 1/2$

$$\frac{dx}{dt} = \pi\cos\pi t, \quad \left(\frac{dx}{dt}\right)^2 = \pi^2\cos^2\pi t$$

$$\frac{dy}{dt} = -\pi\sin\pi t, \quad \left(\frac{dy}{dt}\right)^2 = \pi^2\sin^2\pi t$$

$$L = \int_{-1/2}^{1/2} \sqrt{\pi^2\underbrace{(\cos^2\pi t + \sin^2\pi t)}_{1}}\,dt = \int_{-1/2}^{1/2}\sqrt{\pi^2}\,dt = \int_{-1/2}^{1/2}\pi\,dt = \pi x\Big|_{-1/2}^{1/2} = \frac{1}{2}\pi + \frac{1}{2}\pi = \boxed{\pi}$$

37. $L = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt$ means the derivatives are taken with respect to t

Take derivatives with respect to x instead.

$$L = \int_a^b \sqrt{\left(\frac{dx}{dx}\right)^2 + \left(\frac{dy}{dx}\right)^2}\,dx = \boxed{\int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\,dx}$$

39. $x = a(t - \sin t),\ y = a(1 - \cos t)$

$$\text{Area} = \int_0^{2a\pi} y\,dx$$

$$dx = \left(\frac{dx}{dt}\right)dt = a(1-\cos t)\,dt$$

$$\int_0^{2\pi} a(1-\cos t)a(1-\cos t)\,dt$$

$$a^2\int_0^{2\pi}(1 - 2\cos t + \cos^2 t)\,dt$$

Convert bounds from x's to t's:

$0 = a(t - \sin t)$

$a \neq 0 \quad t - \sin t = 0 \to t = \sin t$ when $t = 0$

$\cancel{a}(t - \sin t) = 2\cancel{a}\pi$

$t - \sin t = 2\pi$

$2\pi - \cancel{\sin 2\pi} = 2\pi$ when $t = 2\pi$

Trig Identity: $\cos 2t = 2\cos^2 t - 1$

$2\cos^2 t = 1 + \cos 2t$

$\cos^2 t = \frac{1}{2} + \frac{1}{2}\cos 2t$

$$a^2\int_0^{2\pi}\left(1 - 2\cos t + \frac{1}{2} + \frac{1}{2}\cos 2t\right)dt = a^2\int_0^{2\pi}\left(\frac{3}{2} - 2\cos t + \frac{1}{2}\cos 2t\right)dt$$

$$a^2\left(\frac{3}{2}t - 2\sin t + \frac{1}{4}\sin 2t\right)\Big|_0^{2\pi} = a^2(3\pi - 0 + 0) = \boxed{3\pi a^2}$$