Parametric Functions (Section 11.1)

$* \ \frac{dy}{dx} = \frac{dy/dt}{dx/dt}$ AND $\frac{d^2y}{dx^2} = \frac{dy'/dt}{dx/dt}$ ← derivative of y' w/respect to t ← only dx/dt

ex: $x = \sin t$
$y = \sqrt{3}\cos t$

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{-\sqrt{3}\sin t}{\cos t} = \boxed{-\sqrt{3}\tan t}$$

$$\frac{d^2y}{dx^2} = \frac{\frac{d}{dt}(-\sqrt{3}\tan t)}{dx/dt} = \frac{-\sqrt{3}\sec^2 t}{\cos t} = \boxed{-\sqrt{3}\sec^3 t}$$

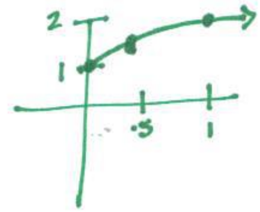
#3 $x = \tan t$
$y = \sec t$
$0 \le t \le \pi/4$

$\sec^2\theta = 1 + \tan^2\theta$ (trig identity)

$y^2 = 1 + x^2$

$y = \sqrt{1+x^2}$

t	x	y
0	0	1
$\pi/8$	.4	1.1
$\pi/4$	1	1.4



$* $ Arc Length: $L = \int_a^b \sqrt{(dx/dt)^2 + (dy/dt)^2}\, dt$

ex: $x = \cos t$
$y = \sqrt{3}\cos t$
$0 \le t \le 2\pi$

$$L = \int_0^{2\pi} \sqrt{(-\sin t)^2 + (-\sqrt{3}\sin t)^2}\, dt$$

$$= \int_0^{2\pi} \sqrt{(\sin^2 t + 3\sin^2 t)}\, dt$$

$$= \int_0^{2\pi} \sqrt{4\sin^2 t}\, dt = \int_0^{2\pi} |2\sin t|\, dt$$

$$= 2\int_0^{\pi} 2\sin t\, dt \ \ \underline{\underline{OR}} \ \ 4\int_0^{\pi/2} 2\sin t\, dt$$

$*$ Sine is (+) in Q1 & Q2

$$= -8\cos t \Big|_0^{\pi/2}$$

$$= -8(0-1) \boxed{= 8}$$

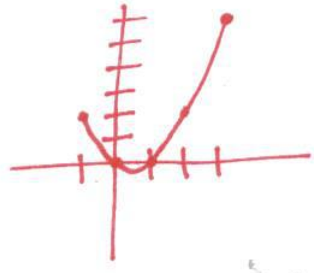
#17 $x = t+1$

$y = t^2+t$

$-2 \le t \le 2$

a)

t	x	y
-2	-1	2
-1	0	0
0		0
1	2	2
2	3	6



b) Lowest Point → Do part c first.

c) $\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{2t+1}{1} = 2t+1$

$2t+1=0$

$2t=-1$

$t=-\frac{1}{2}$

$\boxed{\therefore\ x=\frac{1}{2},\ y=-\frac{1}{4}}$

* Remember: Horz. Line: $m=0 \to \frac{dy}{dt}=0$

Vert Line: $m=\text{undef} \to \frac{dx}{dt}=0$

#23 $x = 2+\cos t$

$y = -1+\sin t$

Horz: $\frac{dy}{dt}=0$

$\frac{dy}{dt} = \cos t = 0$

$t = \frac{\pi}{2}, \frac{3\pi}{2}$

$x = 2+\cos\frac{\pi}{2}$

$y = -1+\sin\frac{\pi}{2}$

$\boxed{(2,0)}$

$x = 2+\cos\frac{3\pi}{2}$

$y = -1+\sin\frac{3\pi}{2}$

$\boxed{(2,-2)}$

Vert: $\frac{dx}{dt}=0$

$\frac{dx}{dt} = -\sin t = 0$

$t = 0, \pi, 2\pi, \ldots$

$x = 2+\cos 0$

$y = -1+\sin 0$

$\boxed{(3,-1)}$

$x = 2+\cos\pi$

$y = -1+\sin\pi$

$\boxed{(1,-1)}$