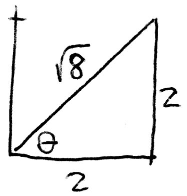
Section 11.2: 1-23 odd, 27-51 e.o.o.

1. $\boxed{\langle 2,3 \rangle}$

3. $\boxed{\langle 1,-4 \rangle}$

5. $\sqrt{2^2+2^2} = \sqrt{4+4} = \boxed{\sqrt{8}}$

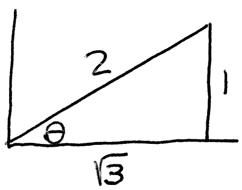


$\sin\theta = \frac{2}{\sqrt{8}} = \frac{2}{2\sqrt{2}} = \frac{1}{\sqrt{2}}$

$\cos\theta = \frac{2}{\sqrt{8}} = \frac{2}{2\sqrt{2}} = \frac{1}{\sqrt{2}}$

$\boxed{\frac{\pi}{4}}$

7. $\sqrt{\sqrt{3}^2 + 1^2} = \sqrt{3+1} = \sqrt{4} = \boxed{2}$

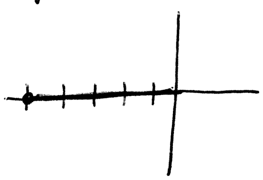


$\sin\theta = \frac{1}{2}$

$\cos\theta = \frac{\sqrt{3}}{2}$

$\boxed{\frac{\pi}{6}}$

9. $\sqrt{(-5)^2+0^2} = \sqrt{25+0} = \sqrt{25} = \boxed{5}$



$\boxed{\pi}$

11. $x = 4\cos\pi = -4$

$y = 4\sin\pi = 0$

$\boxed{\langle -4,0 \rangle}$

13. $x = 5\cos 100^\circ = -0.868$

$y = 5\sin 100^\circ = 4.924$

$\boxed{\langle -0.868, 4.924 \rangle}$

15. $x = 3\sqrt{2}\cos \pi/4 = 3\cancel{\sqrt{2}} \cdot \frac{1}{\cancel{\sqrt{2}}} = 3$

$y = 3\sqrt{2}\sin \pi/4 = 3\cancel{\sqrt{2}} \cdot \frac{1}{\cancel{\sqrt{2}}} = 3$

$\boxed{\langle 3,3 \rangle}$

17. $u = \langle 3,-2 \rangle$, $v = \langle -2,5 \rangle$

a) $3u = 3\langle 3,-2 \rangle = \boxed{\langle 9,-6 \rangle}$

b) $\sqrt{9^2+(-6)^2} = \sqrt{81+36} = \boxed{\sqrt{117} \approx 10.817}$

$u = \langle 3,-2\rangle, \quad v = \langle -2,5\rangle$

19. $u+v = \langle 3,-2\rangle + \langle -2,5\rangle = \boxed{\langle 1,3\rangle}$

$\sqrt{1^2+3^2} = \sqrt{1+9} = \boxed{\sqrt{10} \approx 3.162}$

21. $2u-3v = 2\langle 3,-2\rangle - 3\langle -2,5\rangle = \langle 6,-4\rangle - \langle -6,15\rangle = \boxed{\langle 12,-19\rangle}$

$\sqrt{12^2+(-19)^2} = \sqrt{144+361} = \boxed{\sqrt{505} \approx 22.472}$

23. $\frac{3}{5}u + \frac{4}{5}v = \frac{3}{5}\langle 3,-2\rangle + \frac{4}{5}\langle -2,5\rangle = \langle \frac{9}{5}, \frac{-6}{5}\rangle + \langle \frac{-8}{5}, \frac{20}{5}\rangle = \boxed{\langle \frac{1}{5}, \frac{14}{5}\rangle}$

$\sqrt{(1/5)^2+(14/5)^2} = \sqrt{\frac{1}{25}+\frac{196}{25}} = \sqrt{\frac{197}{25}} = \boxed{\frac{\sqrt{197}}{5} \approx 2.807}$

27. $r(t) = \langle 3t^2, 2t^3\rangle$

$v(t) = \boxed{\langle 6t, 6t^2\rangle}$

$a(t) = \boxed{\langle 6, 12t\rangle}$

31. $r(t) = \langle t^2+\sin 2t,\ t^2-\cos 2t\rangle$

$v(t) = \boxed{\langle 2t+2\cos 2t,\ 2t+2\sin 2t\rangle}$

$a(t) = \boxed{\langle 2-4\sin 2t,\ 2+4\cos 2t\rangle}$

35. $r(t) = \boxed{\langle \cos 3t, \sin 2t\rangle}$ ← path

$v(t) = \boxed{\langle -3\sin 3t, 2\cos 2t\rangle}$

$a(t) = \boxed{\langle -9\cos 3t, -4\sin 2t\rangle}$

39. $v(t) = \langle 3t^2-2t,\ 1+\cos \pi t\rangle$; $(2,6)$ at $t=0$

a) $x = 2 + \int_0^3 (3t^2-2t)\,dt = 2 + (t^3-t^2)\Big|_0^3 = 2+27-9 = 20$

$y = 6 + \int_0^3 (1+\cos \pi t)\,dt = 6 + \left(t + \frac{1}{\pi}\sin \pi t\right)\Big|_0^3 = 6+3+0 = 9$

$\boxed{(20,9)}$

b) Total Distance $= \int_0^3 \sqrt{(3t^2-2t)^2+(1+\cos \pi t)^2}\,dt = \boxed{19.343}$

43. $v(t) = \langle 3t^2 - 2t,\ 1 + \cos \pi t \rangle$

$r(t) = \langle t^3 - t^2 + C,\ t + \frac{1}{\pi}\sin \pi t + C \rangle$

$r(t) = \langle 2, 6 \rangle$ when $t = 0 \rightarrow$ use to find C values

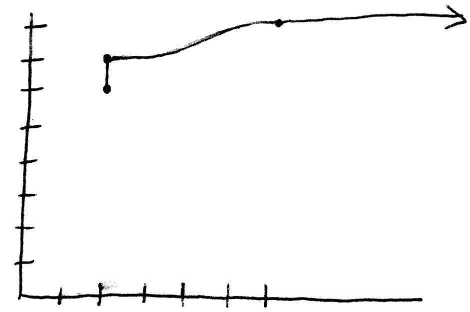
For x, $C = 2$; For y, $C = 6$

$r(t) = \langle t^3 - t^2 + 2,\ t + \frac{1}{\pi}\sin \pi t + 6 \rangle$

Graph $r(t)$ in parametric mode:

$0 \le t \le 3$, Tstep $= 0.1$

$0 \le x \le 20$, $0 \le y \le 10$



47. $r(t) = \left\langle \dfrac{1-t^2}{1+t^2}, \dfrac{2t}{1+t^2} \right\rangle$

a) $v(t) = \left\langle \dfrac{(1+t^2)(-2t) - (1-t^2)2t}{(1+t^2)^2}, \dfrac{(1+t^2)2 - 2t(2t)}{(1+t^2)^2} \right\rangle$

$v(t) = \left\langle \dfrac{-2t - \cancel{2t^3} - 2t + \cancel{2t^3}}{(1+t^2)^2}, \dfrac{2 + 2t^2 - 4t^2}{(1+t^2)^2} \right\rangle = \boxed{\left\langle \dfrac{-4t}{(1+t^2)^2}, \dfrac{2-2t^2}{(1+t^2)^2} \right\rangle}$

b) There is no movement in the x direction when $t=0$, but the y is still increasing then. There is no movement in the y direction when $t=1$, but the x is still decreasing then. For any t, the particle is moving in at least one direction, so the particle is $\boxed{\text{never}}$ at rest.

c) $\displaystyle\lim_{t\to\infty} \frac{1-t^2}{1+t^2} = \lim_{t\to\infty} \frac{-\cancel{t^2}}{\cancel{t^2}} = -1$ is what x approaches

$\displaystyle\lim_{t\to\infty} \frac{2t}{1+t^2} = \lim_{t\to\infty} \frac{2\cancel{t}}{t^{\cancel{2}}} = \lim_{t\to\infty} \frac{2}{t} = \frac{2}{\infty} = 0$ is what y approaches

As $t \to \infty$, the particle approaches $\boxed{(-1, 0)}$.

51. $\frac{dx}{dt} = 2 + \sin t^2$

$t = 2: \quad (3,5)$

a) $x = 3 + \int_2^4 (2 + \sin t^2)\,dt = \boxed{6.942}$

b) $\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{-6}{2+\sin 4} \approx -4.826$ slope

$\boxed{y - 5 = -4.826(x-3)}$

c) speed $= \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} = \sqrt{(2+\sin t^2)^2 + (-6)^2} = \sqrt{(2+\sin 4)^2 + 36} \approx \boxed{6.127}$

d) $\frac{dy}{dx} = \frac{dy/dt}{dx/dt} \rightarrow \frac{dy}{dt} = \frac{dy}{dx} \cdot \frac{dx}{dt} = (2t-1)(2+\sin t^2)$

$a(t) = \langle 2t\cos t^2, (2t-1)(2t\cos t^2) + (2+\sin t^2)(2) \rangle$

$a(4) = \langle 8\cos 16, 56\cos 16 + 4 + 2\sin 16 \rangle = \boxed{\langle -7.661, -50.205 \rangle}$