Section 11.2 – Introduction to Vectors

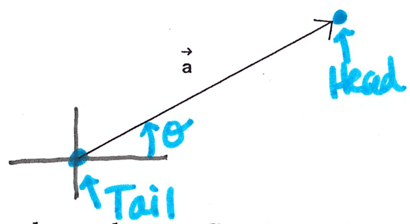
vector $\mathbf{v} \rightarrow \mathbf{v} = \langle a, b \rangle$

- A vector quantity has direction (θ) and magnitude. There are many examples of vector quantities in the natural world, such as force, velocity, and acceleration.

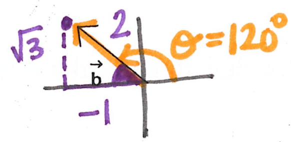
  $\rightarrow$ length: $|\mathbf{v}| = \sqrt{a^2 + b^2}$

- A vector is a directed line segment used to represent a vector quantity.

  Examples:



ex: $(-1, \sqrt{3})$ Find direction & magnitude



$\theta = 120^\circ, \frac{2\pi}{3}$ $\quad |\mathbf{v}| = 2$

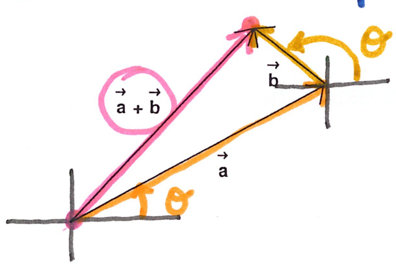
$\theta = \tan^{-1}\left(\frac{\sqrt{3}}{-1}\right)$ $\quad \boxed{\theta = 1.047}$

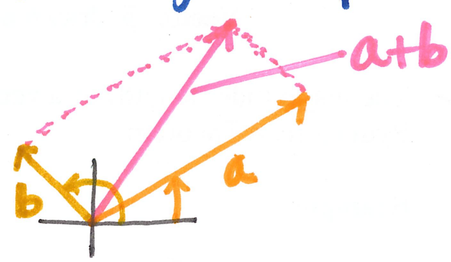
$|\mathbf{v}| = \sqrt{(-1)^2 + (\sqrt{3})^2} = \sqrt{1+3} = \sqrt{4} = \boxed{2}$

- Vectors do not have a fixed position in the coordinate plane, so they can be translated (moved) without changing their meaning.

- To add vectors geometrically, translate the second vector so that it starts at the end of the first vector. The vector sum goes from the start of the first vector to the end of the second. Don't forget to draw the arrow!

  Example: Head to Tail Representation | Parallelogram Representation





- Vectors can be represented numerically. In this class we will write the numeric form of a vector in two ways:

  $\vec{a} = \begin{pmatrix} 3 \\ 2 \\ 7 \end{pmatrix}$ $\qquad$ $\vec{b} = 4\vec{i} - 5\vec{j} + 2\vec{k} \rightarrow \mathbf{b} = 4\mathbf{i} - 5\mathbf{j} + 2\mathbf{k}$

  $\mathbf{b} = \langle 4, -5, 2 \rangle$

  column form **IB** $\qquad$ component form. *AP*

- To add vectors numerically, just add the corresponding components.

  Example: Add vectors $\vec{v} = 3\vec{i} - 4\vec{j}$ and $\vec{w} = 5\vec{i} + 2\vec{j}$.

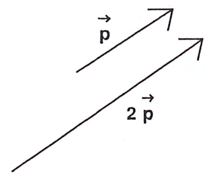
  $\mathbf{v} + \mathbf{w} = (3+5)\mathbf{i} + (-4+2)\mathbf{j}$

  $\boxed{= 8\mathbf{i} - 2\mathbf{j}}$

- A scalar is a number. It has magnitude but no direction. Vectors can be multiplied by scalars.

just distribute the scalar into the vector.

Examples:



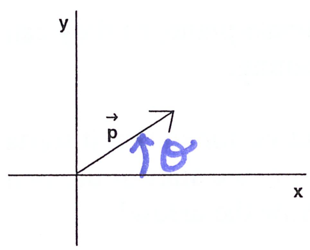
$\vec{m} = -6\vec{i} + 2\vec{j} + 5\vec{k}$

$\frac{1}{2}\vec{m} = -3\vec{i} + \vec{j} + \frac{5}{2}\vec{k}$

ex:
$3m = -18i + 6j + 15k$

- A position vector is a vector that starts at the origin (0, 0).

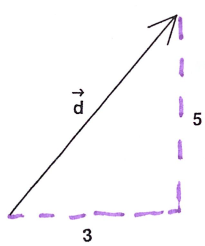
Example:



Vector $\vec{p}$ drawn as a position vector.

- The magnitude (length) of a vector can be calculated using the Pythagorean Theorem. $\rightarrow$ $|d| = \sqrt{a^2+b^2}$

Example:



$\vec{d} = 3\vec{i} + 5\vec{j}$

$|\vec{d}| = \sqrt{3^2 + 5^2}$

$= \sqrt{9+25}$

$= \sqrt{34}$ $\rightarrow$ scalar!

Example: Find the length of $\vec{c} = -2\vec{i} - 4\vec{j}$.

$|\vec{c}| = \sqrt{(-2)^2 + (-4)^2}$

$= \sqrt{4+16}$

$= \sqrt{20}$

$|c| = \sqrt{(-2)^2 + (-4)^2}$

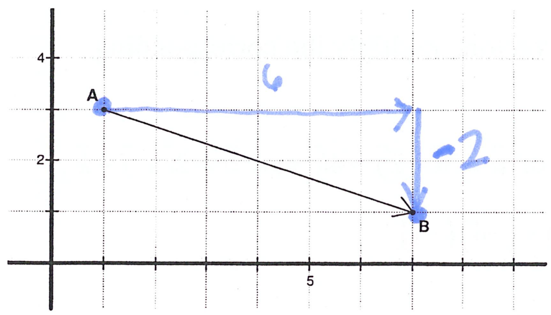
$= \sqrt{4+16}$

$\boxed{= \sqrt{20}}$

- A displacement vector is a vector between two points.

position = starting point + displacement

Example: Draw the displacement vector from $A(1, 3)$ to $B(7, 1)$. Then write vector $\overrightarrow{AB}$ in component form.



$\overrightarrow{AB} = \langle (7-1), (1-3) \rangle$

$= \langle 6, -2 \rangle$

$\cancel{\text{Vector } \overrightarrow{AB} = 6\vec{i} - 2\vec{j}}$

- To find a displacement vector numerically, subtract the coordinates:
  "end point minus start point"

Example: Find the displacement vector from $C(-3, 5, -1)$ to $D(4, -2, 7)$.

$$\overrightarrow{CD} = (4-(-3))\vec{i} + (-2-5)\vec{j} + (7-(-1))\vec{k}$$

$$\boxed{= 7\vec{i} - 7\vec{j} + 8\vec{k}}$$

- A unit vector is a vector that is one unit long. magnitude = 1

unit vector $= \frac{v}{|v|}$

Example: Determine whether or not each of the following vectors is a unit vector:

a) $\vec{v} = \vec{i} + \vec{j}$ b) $\vec{w} = \frac{1}{2}\vec{i} + \frac{1}{2}\vec{j}$ c) $\vec{m} = \frac{3}{5}\vec{i} + \frac{4}{5}\vec{j}$

$|\vec{v}| = \sqrt{1^2 + 1^2} = \sqrt{2} \approx 1.4$ $\vec{v}$ is not a unit vector.

$|\vec{w}| = \sqrt{\left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2} = \sqrt{\frac{1}{2}} \approx 0.7$ $\vec{w}$ is not a unit vector.

$|\vec{m}| = \sqrt{\left(\frac{3}{5}\right)^2 + \left(\frac{4}{5}\right)^2} = \sqrt{1} = 1$ $\vec{m}$ is a unit vector. ✓

- To find a unit vector parallel to a given vector, divide the vector by its length.

Example: Find a unit vector parallel to $\vec{a} = 3\vec{i} + 8\vec{j}$

$|a| = \sqrt{3^2 + 8^2} = \sqrt{9+64}$

$= \sqrt{73}$

$\cancel{|\vec{a}| = \sqrt{3^2 + 8^2} = \sqrt{73}}$

$\cancel{\vec{u} = \frac{\vec{a}}{|\vec{a}|} = \frac{3}{\sqrt{73}}\vec{i} + \frac{8}{\sqrt{73}}\vec{j}}$

unit vector a: $\left\langle \frac{3}{\sqrt{73}}, \frac{8}{\sqrt{73}} \right\rangle$

Chapters 15 and 16 – The Dot Product

- One way of multiplying two vectors is to find the dot product. The dot product of vectors $\vec{a}$ and $\vec{b}$ is written $\vec{a}\cdot\vec{b}$.

- To find the dot product of two vectors, multiply the corresponding components and add them up.

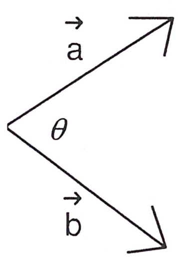
Example: Find the dot product of $\vec{a}=3\vec{i}-2\vec{j}-4\vec{k}$ and $\vec{b}=2\vec{i}+\vec{j}-\vec{k}$.

$$\cancel{\vec{a}\cdot\vec{b}=3(2)+(-2)(1)+(-4)(-1)}$$
$$\cancel{=6-2+4=8}$$

$$a\cdot b = 3(2)+(-2)(1)+(-4)(-1)$$
$$6+-2+4 \boxed{=8} \leftarrow \text{scalar}$$

- The dot product is also called the scalar product because the result is a scalar, not a vector.

- The dot product is used to find the angle between two vectors when they are placed tail-to-tail. The formula is:



$$\boxed{\cos\theta=\frac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|}}$$

$$\theta=\cos^{-1}\left(\frac{a\cdot b}{|a|\cdot|b|}\right)$$

→ vector a "dot" vector b

magn. of a times magn of b.

Example: Find the angle (in degrees) between $\vec{c}=\begin{pmatrix}3\\1\\7\end{pmatrix}$ and $\vec{d}=\begin{pmatrix}-2\\5\\1\end{pmatrix}$.

$$\cancel{\vec{c}\cdot\vec{d}=3(-2)+1(5)+7(1)=6}$$
$$\cancel{|\vec{c}|=\sqrt{3^2+1^2+7^2}=\sqrt{59}}$$
$$\cancel{|\vec{d}|=\sqrt{(-2)^2+5^2+1^2}=\sqrt{30}}$$
$$\cancel{\cos\theta=\frac{6}{\sqrt{59}\cdot\sqrt{30}}\approx 0.143}$$
$$\cancel{\theta=\cos^{-1}(0.143)\approx 81.801^\circ}$$

$$c\cdot d = 3(-2)+1(5)+7(1)$$
$$=-6+5+7=6$$
$$|c|=\sqrt{3^2+1^2+7^2}=\sqrt{9+1+49}=\sqrt{59}$$
$$|d|=\sqrt{(-2)^2+5^2+1^2}=\sqrt{4+25+1}=\sqrt{30}$$
$$\theta=\cos^{-1}\left(\frac{6}{\sqrt{59}\cdot\sqrt{30}}\right)=\boxed{81.8^\circ}$$