Normal, Complimentary, "orthogonal"

- Vectors are perpendicular when the angle between them is a right angle. Numerically, vectors are perpendicular if and only if their dot product equals zero (since $\cos 90^\circ = 0$).

Example: Show that vectors $\vec{g} = 2\vec{i} - 15\vec{j} - \vec{k}$ and $\vec{h} = 6\vec{i} + \vec{j} - 3\vec{k}$ are perpendicular.

$$\vec{g}\cdot\vec{h} = 2(6) + (-15)(1) + (-1)(-3) = 0 \checkmark$$

Example: Find the value of k so that vectors $\vec{a} = \begin{pmatrix} 1 \\ 1 \\ -3 \end{pmatrix}$ and $\vec{b} = \begin{pmatrix} 1 \\ -10 \\ k \end{pmatrix}$ are perpendicular.

$$\cancel{\vec{a}\cdot\vec{b} = 1(1) + (1)(-10) + (-3)(k) = -9 - 3k}$$

$$\cancel{-9 - 3k = 0}$$

$$\cancel{-3k = 9}$$

$$\cancel{k = -3}$$

$a \cdot b = 1(1) + 1(-10) + -3(k)$

$0 = 1 - 10 - 3k$

$9 = -3k$

$\boxed{k = -3}$

- Vectors are parallel when they have the same direction (or opposite directions). Numerically, vectors are parallel if and only if they are scalar multiples of each other.

Example: Find the value of k so that vectors $\vec{c} = \begin{pmatrix} -12 \\ -8 \\ k \end{pmatrix}$ and $\vec{d} = \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}$ are parallel.

-4 -4 -4

$\boxed{k = -4}$

$$-12 = 3(-4) \text{ and } -8 = 2(-4) \text{ so } k = 1(-4) = -4$$

Chapters 17 – Vector Equations of Lines **IB**

- You already know how to write the Cartesian equation, $y = mx + b$, of a line if you know the slope and any point on the line.

- Lines can also be defined by vector equations. To write the vector equation of a line you need to know a point on the line and the direction vector for the line (a vector parallel to the line).

- The general form of the vector equation of a line is

$$\vec{r} = \vec{a} + t\vec{b}$$

a → point
b → slope

where $\vec{a}$ is the position vector to a point on the line, $\vec{b}$ is the direction vector for the line, t is an arbitrary scalar (the independent variable) and $\vec{r} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$ (the dependent variable).

Example: Write a vector equation of the line passing through point $(2, -1)$ and parallel to $3\vec{i} - 5\vec{j}$.

$$\vec{r} = \begin{pmatrix} 2 \\ -1 \end{pmatrix} + t\begin{pmatrix} 3 \\ -5 \end{pmatrix}$$

slope

Example: Write a vector equation of the line passing through $(-1, 3, 7)$ and parallel to $5\vec{i} - 2\vec{j} + \vec{k}$.

$$\vec{r} = \begin{pmatrix} -1 \\ 3 \\ 7 \end{pmatrix} + t\begin{pmatrix} 5 \\ -2 \\ 1 \end{pmatrix}$$

Example: Write a vector equation of the line passing through the points $A(2, -1, 5)$ and $B(3, 1, 2)$

*must use the starting point.

~~a direction vector is~~ $\cancel{\overrightarrow{AB} = \begin{pmatrix} 1 \\ 2 \\ -3 \end{pmatrix}}$

~~so an equation is~~ $\cancel{\vec{r} = \begin{pmatrix} 2 \\ -1 \\ 5 \end{pmatrix} + t\begin{pmatrix} 1 \\ 2 \\ -3 \end{pmatrix}}$

$(B-A)$

$\overline{AB} = \langle 3-2, 1-1, 2-5 \rangle$

$= \langle 1, 2, -3 \rangle$

Slope

$$\boxed{r = \begin{pmatrix} 2 \\ -1 \\ 5 \end{pmatrix} + t\begin{pmatrix} 1 \\ 2 \\ -3 \end{pmatrix}}$$

- When you know the vector equation of a line, you can find points on the line by choosing any number for t and plugging it in to the equation.

Example: Write down three points on the line $\vec{r} = \begin{pmatrix}1\\5\\2\end{pmatrix} + t\begin{pmatrix}3\\-1\\1\end{pmatrix}$

~~when $t = 1$, the point is (4, 4, 3)~~

~~when $t = -1$, the point is (–2, 6, 1)~~

~~when $t = 2$, the point is (7, 3, 4)~~

$t=3$ $\quad r = \begin{pmatrix}1\\5\\2\end{pmatrix} + 3\begin{pmatrix}3\\-1\\1\end{pmatrix} = \begin{pmatrix}1\\5\\2\end{pmatrix} + \begin{pmatrix}9\\-3\\3\end{pmatrix}$

$\boxed{= \begin{pmatrix}10\\2\\5\end{pmatrix}}$

(Remember these are just a few examples – you can pick any number you want for t!)

- If two vector lines intersect, you can find the point of intersection by setting the equations equal to each other, solving for one of the independent variables, and then plugging back in to the equation to get the coordinates of the point.

Solved for t

Example: The lines $\boxed{\vec{r}_1 = \begin{pmatrix}-2\\-3\end{pmatrix} + t\begin{pmatrix}3\\-2\end{pmatrix}}$ and $\vec{r}_2 = \begin{pmatrix}6\\5\end{pmatrix} + s\begin{pmatrix}-2\\-4\end{pmatrix}$ intersect at the point P. Find the coordinates of P.

STEPS

1. Take it out of vector form.
2. 2 Equations, 2 Unknowns
3. Elimination/Substitution to solve for a variable.
4. Plug that value into the appropriate equation to get x & y.

1 $\quad -2+3t = 6-2s$ (x)

$\quad -3-2t = 5-4s$ (y)

$\rightarrow$

$-4+6t = 12-4s$ 2

$-(-3-2t = 5-4s)$

$-1+8t = 7$

$8t = 8$

$t = 1$ 3

$x = -2+1(3) = 1$

$y = -3+1(-2) = -5$

$\boxed{\text{The coordinates of } \underline{P} \text{ are } (1, -5)}$

$*\ x = |v| \cdot \cos\theta$

$\ \ \ y = |v| \cdot \sin\theta$

* Position Vector:

$r(t) = \langle x(t), y(t) \rangle$

* Velocity Vector: $v(t) = \langle \frac{dx}{dt}, \frac{dy}{dt} \rangle$

* Acceleration Vector: $a(t) = \langle \frac{d^2x}{dt^2}, \frac{d^2y}{dt^2} \rangle$

* Speed: $|v(t)| \rightarrow$ magnitude $\rightarrow$ scalar

* **Displacement** from $t=a$ to $t=b$ of $\mathbf{v}(t) = \langle v_1(t), v_2(t) \rangle$:

$$= \left\langle \int_a^b v_1(t)\,dt, \int_a^b v_2(t)\,dt \right\rangle$$

* **Distance Traveled** from $t=a$ to $t=b$ of $\mathbf{v}(t)$:

$$= \int_a^b |\mathbf{v}(t)|\,dt = \int_a^b \sqrt{[v_1(t)]^2 + [v_2(t)]^2}\,dt$$

ex: $\mathbf{v}(t) = \langle 3t^2 - 2t, 1 + \cos \pi t \rangle$; starts @ $(2,6)$ from $t=0$ to $t=3$.

displacement: $\boxed{\langle 18, 3 \rangle}$

$$\int_0^3 (3t^2 - 2t)\,dt = t^3 - t^2 \Big|_0^3 = 3^3 - 3^2 - (0) = 18$$

$$\int_0^3 (1 + \cos \pi t)\,dt = t + \frac{\sin \pi t}{\pi} \Big|_0^3 = 3 + \frac{\sin 3\cancel{\pi}}{\cancel{\pi}} - \left(\cancel{0} + \frac{\sin 0}{\pi}\right) = 3$$

position: starting point + displacement $= \langle 2, 6 \rangle + \langle 18, 3 \rangle$ $\boxed{= \langle 20, 9 \rangle}$

distance traveled: $\int_0^3 \sqrt{(\underbrace{3t^2 - 2t}_{y_1})^2 + (\underbrace{1 + \cos \pi t}_{y_2})^2}\,dt = \boxed{19.343}$