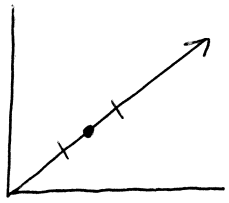
Section 11.3: 1-41 odd

1. a) $(\sqrt{2}, \pi/4)$

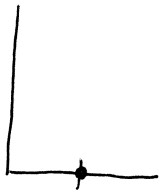


$x = \sqrt{2}\cos\frac{\pi}{4} = \sqrt{2}\cdot\frac{1}{\sqrt{2}} = 1$

$y = \sqrt{2}\sin\frac{\pi}{4} = \sqrt{2}\cdot\frac{1}{\sqrt{2}} = 1$

$\boxed{(1,1)}$

b) $(1,0)$

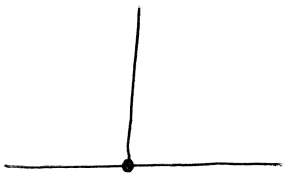


$x = 1\cos 0 = 1\cdot 1 = 1$

$y = 1\sin 0 = 1\cdot 0 = 0$

$\boxed{(1,0)}$

c) $(0, \pi/2)$

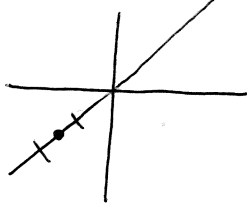


$x = 0\cos\pi/2 = 0\cdot 0 = 0$

$y = 0\sin\pi/2 = 0\cdot 1 = 0$

$\boxed{(0,0)}$

d) $(-\sqrt{2}, \pi/4)$

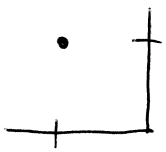


$x = -\sqrt{2}\cos\frac{\pi}{4} = -\sqrt{2}\cdot\frac{1}{\sqrt{2}} = -1$

$y = -\sqrt{2}\sin\frac{\pi}{4} = -\sqrt{2}\cdot\frac{1}{\sqrt{2}} = -1$

$\boxed{(-1,-1)}$

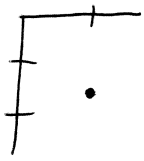
3. a) $(-1,1)$



$r = \sqrt{1^2+1^2} = \sqrt{2}$

$\boxed{(\sqrt{2}, 3\pi/4)}$
$\boxed{(-\sqrt{2}, -\pi/4)}$

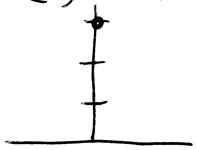
b) $(1,-\sqrt{3})$



$r = \sqrt{1+3} = \sqrt{4} = 2$

$\boxed{(2, 5\pi/3)}$
$\boxed{(-2, 2\pi/3)}$

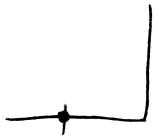
c) $(0,3)$



$r = \sqrt{0+9} = \sqrt{9} = 3$

$\boxed{(3, \pi/2)}$
$\boxed{(-3, 3\pi/2)}$

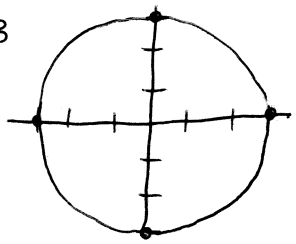
d) $(-1,0)$



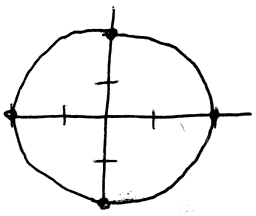
$r = \sqrt{1+0} = \sqrt{1} = 1$

$\boxed{(1, \pi)}$
$\boxed{(-1, 0)}$

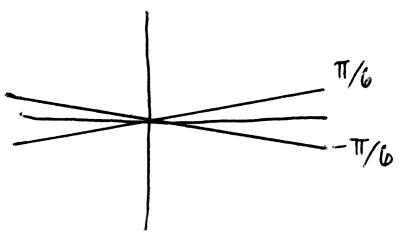
5. $r = 3$
Circle



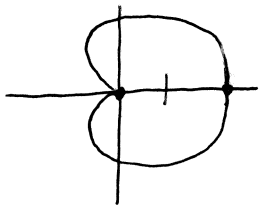
7. $r^2 = 4 \rightarrow r = \pm 2$



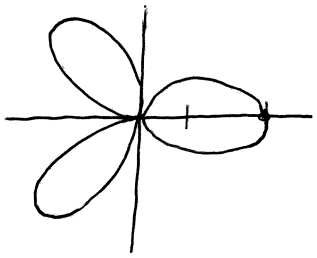
9. $|\theta| = \frac{\pi}{6} \rightarrow \theta = \pm\frac{\pi}{6}$



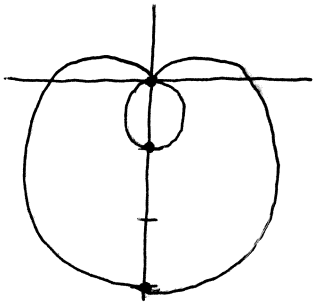
11. $r = 1 + \cos\theta$
Cardiod



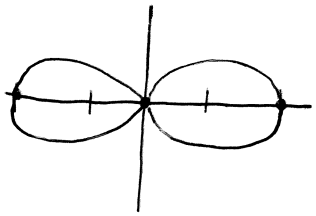
13. $r = 2\cos 3\theta$
Rose



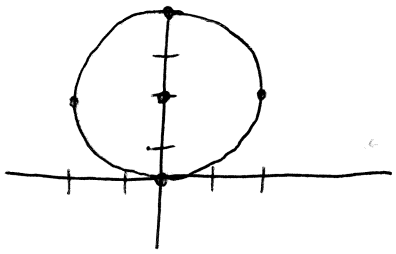
15. $r = 1 - 2\sin\theta$
Limacon



17. $r^2 = 4\cos 2\theta$
$r = \sqrt{4\cos 2\theta} = 2\sqrt{\cos 2\theta}$
Lemniscate



19. $r = 4\sin\theta$
Circle



21. $r = 4\csc\theta \rightarrow r = \frac{4}{\sin\theta} \rightarrow r\sin\theta = 4 \rightarrow \boxed{y = 4}$ (Horizontal Line)

23. $r\cos\theta + r\sin\theta = 1 \rightarrow x + y = 1 \rightarrow \boxed{y = -x + 1}$ (Line w/ slope $= -1$, y-int $= 1$)

25. $r = \frac{5}{\sin\theta - 2\cos\theta} \rightarrow r\sin\theta - 2r\cos\theta = 5 \rightarrow y - 2x = 5 \rightarrow \boxed{y = 2x + 5}$
(Line w/ slope $= 2$, y-int $= 5$)

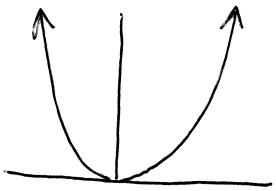
27. $\cos^2\theta = \sin^2\theta \rightarrow r^2\cos^2\theta = r^2\sin^2\theta \rightarrow x^2 = y^2 \rightarrow y = \pm\sqrt{x^2} \rightarrow \boxed{y = \pm x}$

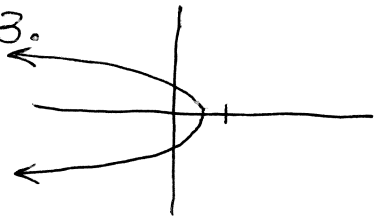
(Two lines: $y = x$ and $y = -x$)

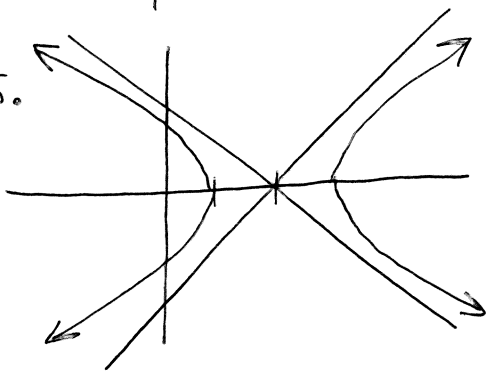
29. $r = 8\sin\theta$

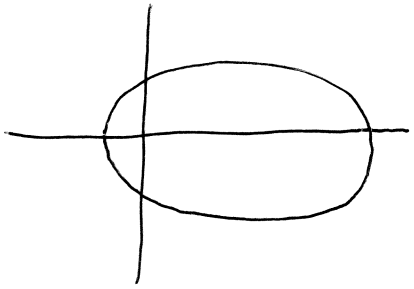
$r^2 = 8r\sin\theta \rightarrow x^2 + y^2 = 8y \rightarrow x^2 + y^2 - 8y = 0$

$x^2 + y^2 - 8y + 16 = 16 \rightarrow \boxed{x^2 + (y-4)^2 = 16}$ (Circle w/ center $(0,4)$ & $r = 4$)

31.  Parabola

33.  Parabola

35.  Hyperbola

37.  Ellipse

39. $r = -1 + \sin\theta \rightarrow x = r\cos\theta = (-1+\sin\theta)\cos\theta = -\cos\theta + \sin\theta\cos\theta$

$y = r\sin\theta = (-1+\sin\theta)\sin\theta = -\sin\theta + \sin^2\theta$

$$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{-\cos\theta + 2\sin\theta\cos\theta}{\sin\theta + \sin\theta(-\sin\theta) + \cos\theta(\cos\theta)}$$

$\frac{dy}{dx}$ at $\theta = 0$, so $x = 1$, $y = 0$: $\frac{-1+0}{0+0+1} = \frac{-1}{1} = \boxed{-1}$

$\frac{dy}{dx}$ at $\theta = \pi$, so $x = -1$, $y = 0$: $\frac{1+0}{0+0+1} = \frac{1}{1} = \boxed{1}$

41. $r = 2 - 3\sin\theta$

$x = r\cos\theta = (2 - 3\sin\theta)\cos\theta = 2\cos\theta - 3\sin\theta\cos\theta$

$y = r\sin\theta = (2 - 3\sin\theta)\sin\theta = 2\sin\theta - 3\sin^2\theta$

$$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{2\cos\theta - 6\sin\theta\cos\theta}{-2\sin\theta + 3\sin^2\theta - 3\cos^2\theta}$$

$\frac{dy}{dx}$ at $\theta = 0$, so $\begin{matrix} x=1 \\ y=0 \end{matrix}$: $\frac{2 - 0}{0 + 0 - 3} = \frac{2}{-3} = \boxed{\frac{-2}{3}}$

$\frac{dy}{dx}$ at $\theta = \frac{\pi}{2}$, so $\begin{matrix} x=0 \\ y=1 \end{matrix}$: $\frac{0 - 0}{-2 + 3 - 0} = \frac{0}{1} = \boxed{0}$

$\frac{dy}{dx}$ at $\theta = \pi$, so $\begin{matrix} x=-1 \\ y=0 \end{matrix}$: $\frac{-2 - 0}{0 + 0 - 3} = \frac{-2}{-3} = \boxed{\frac{2}{3}}$

$\frac{dy}{dx}$ at $\theta = \frac{3\pi}{2}$, so $\begin{matrix} x=0 \\ y=-1 \end{matrix}$: $\frac{0 - 0}{2 + 3 - 0} = \frac{0}{5} = \boxed{0}$