Section 2.4: 1-29 odd, 33-37 all

1. $f(x) = x^3+1$

a) $[2,3]$

$f(2) = 2^3+1 = 9$

$f(3) = 3^3+1 = 28$

$$\frac{28-9}{3-2} = \frac{19}{1} = \boxed{19}$$

b) $[-1,1]$

$f(-1) = (-1)^3+1 = 0$

$f(1) = 1^3+1 = 2$

$$\frac{2-0}{1--1} = \frac{2}{2} = \boxed{1}$$

3. $f(x) = e^x$

a) $[-2,0]$

$f(-2) = e^{-2} = \frac{1}{e^2}$

$f(0) = e^0 = 1$

$$\frac{1-1/e^2}{0--2} = \boxed{\frac{1-1/e^2}{2}}$$

b) $[1,3]$

$f(1) = e^1 = e$

$f(3) = e^3$

$$\frac{e^3-e}{3-1} = \boxed{\frac{e^3-e}{2}}$$

5. $f(x) = \cot x = \frac{\cos x}{\sin x}$

a) $[\pi/4, 3\pi/4]$

$f(\pi/4) = \frac{\sqrt{2}/2}{\sqrt{2}/2} = 1$

$f(3\pi/4) = \frac{-\sqrt{2}/2}{\sqrt{2}/2} = -1$

$$\frac{-1-1}{\frac{3\pi}{4}-\frac{\pi}{4}} = \frac{-2}{\pi/2} = \boxed{\frac{-4}{\pi}}$$

b) $[\pi/6, \pi/2]$

$f(\pi/6) = \frac{\sqrt{3}/2}{1/2} = \sqrt{3}$

$f(\pi/2) = \frac{0}{1} = 0$

$$\frac{0-\sqrt{3}}{\frac{\pi}{2}-\frac{\pi}{6}} = \frac{-\sqrt{3}}{\pi/3} = \boxed{\frac{-3\sqrt{3}}{\pi}}$$

7. Units for speed: m/s

Estimate points: $Q_1(10,225)$, $Q_2(14,375)$, $Q_3(16.5,475)$, $Q_4(18,550)$, $P(20,650)$

a) $PQ_1 = \frac{650-225}{20-10} = 42.5$ m/s

$PQ_2 = \frac{650-375}{20-14} = 45.8$ m/s

$PQ_3 = \frac{650-475}{20-16.5} = 50$ m/s

$PQ_4 = \frac{650-550}{20-18} = 50$ m/s

Secant	Slope
PQ_1	42.5
PQ_2	45.8
PQ_3	50
PQ_4	50

b) Speed ≈ 50 m/s

9. $y = x^2$ at $x = -2$ $\quad (-2,4)$

a) $\lim\limits_{h\to 0} \dfrac{f(-2+h)-f(-2)}{h} = \lim\limits_{h\to 0} \dfrac{(-2+h)^2-(-2)^2}{h} = \lim\limits_{h\to 0} \dfrac{\cancel{4}-4h+h^2-\cancel{4}}{h} = \lim\limits_{h\to 0} -4+h = -4+0 = \boxed{-4}$

b) $y-4 = -4(x+2) \to y-4 = -4x-8 \to \boxed{y = -4x-4}$

c) $m = 1/4 \to y-4 = \frac{1}{4}(x+2) \to y-4 = \frac{1}{4}x+\frac{1}{2} \to \boxed{y = \frac{1}{4}x+\frac{9}{2}}$

d) On calculator

11. $y = \dfrac{1}{x-1}$ at $x=2$ $\quad (2,1)$

a) $\lim\limits_{h\to 0} \dfrac{f(a+h)-f(a)}{h} = \lim\limits_{h\to 0} \dfrac{\frac{1}{2+h-1}-\frac{1}{2-1}}{h} = \lim\limits_{h\to 0} \dfrac{\frac{1}{h+1}-\frac{1}{1}}{h} = \lim\limits_{h\to 0} \dfrac{\frac{1}{h+1}-\frac{h+1}{h+1}}{h}$

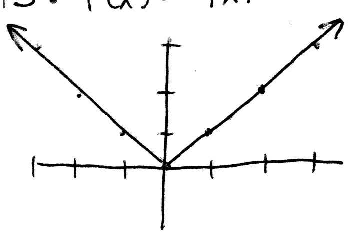
$\lim\limits_{h\to 0} \dfrac{\frac{\cancel{1}-h-\cancel{1}}{h+1}}{h} = \lim\limits_{h\to 0} \dfrac{\frac{-\cancel{h}}{h+1}}{\cancel{h}} = \lim\limits_{h\to 0} \dfrac{-1}{h+1} = \dfrac{-1}{0+1} = \boxed{-1}$

b) $y-1 = -1(x-2) \to y-1 = -x+2 \to \boxed{y = -x+3}$

c) $m = 1 \to y-1 = 1(x-2) \to y-1 = x-2 \to \boxed{y = x-1}$

d) On calculator

13. $f(x) = |x|$



a) $x=2$, $\boxed{m=1}$

b) $x=-3$, $\boxed{m=-1}$

15. $f(x) = \begin{cases} 2-2x-x^2, & x<0 \\ 2x+2, & x \geq 0 \end{cases}$

$\lim\limits_{h\to 0} \dfrac{f(\cancel{0}+h)-f(0)}{h} = \lim\limits_{h\to 0} \dfrac{\cancel{2}-2h-h^2-\cancel{2}}{h} = \lim\limits_{h\to 0} -2-h = -2-0 = -2$ from left

$\lim\limits_{h\to 0} \dfrac{f(\cancel{0}+h)-f(0)}{h} = \lim\limits_{h\to 0} \dfrac{2h+\cancel{2}-\cancel{2}}{h} = 2$ from right

There is no tangent bc slope from left $\neq$ slope from right.

17. $f(x) = \begin{cases} 1/x, & x \le 2 \\ -\frac{1}{4}x+1, & x>2 \end{cases}$

$$\lim_{h\to 0} \frac{f(2+h)-f(2)}{h} = \lim_{h\to 0} \frac{\frac{1}{2+h} - \frac{1}{2}}{h} = \lim_{h\to 0} \frac{\frac{2-(2+h)}{2(2+h)}}{h} = \lim_{h\to 0} \frac{\frac{-\cancel{h}}{4+2h}}{\cancel{h}}$$

$$\lim_{h\to 0} \frac{-1}{4+2h} = \frac{-1}{4+0} = \frac{-1}{4} \text{ from left}$$

$$\lim_{h\to 0} \frac{f(2+h)-f(2)}{h} = \lim_{h\to 0} \frac{-1/4(2+h)+\cancel{1}+1/4(2)-\cancel{1}}{h} = \lim_{h\to 0} \frac{-\cancel{1/2}-1/4h+\cancel{1/2}}{h} = \frac{-1}{4} \text{ from right}$$

Yes, there is a tangent with $m = -1/4$ bc slope from left = slope from right.

19. $y = x^2+2$

a) $$\lim_{h\to 0} \frac{f(a+h)-f(a)}{h} = \lim_{h\to 0} \frac{(a+h)^2+\cancel{2}-a^2-\cancel{2}}{h} = \lim_{h\to 0} \frac{\cancel{a^2}+2ah+h^2-\cancel{a^2}}{h} = \lim_{h\to 0} 2a+h$$
$$= 2a+0 = \boxed{2a}$$

b) As a increases, the slope increases.

21. $y = \frac{1}{x-1}$

a) $$\lim_{h\to 0} \frac{f(a+h)-f(a)}{h} = \lim_{h\to 0} \frac{\frac{1}{a+h-1} - \frac{1}{a-1}}{h} = \lim_{h\to 0} \frac{\frac{\cancel{a}-\cancel{1}-(\cancel{a}+h-\cancel{1})}{(a-1)(a+h-1)}}{h} = \lim_{h\to 0} \frac{\frac{-\cancel{h}}{(a-1)(a+h-1)}}{\cancel{h}}$$

$$\lim_{h\to 0} \frac{-1}{(a-1)(a+h-1)} = \frac{-1}{(a-1)(a+\cancel{a}-1)} = \boxed{\frac{-1}{(a-1)^2}}$$

b) The slopes are always negative. As a increases, the slope approaches 0

23. $f(t) = 3t-7,\ t=1$

$$\lim_{h\to 0} \frac{f(1+h)-f(1)}{h} = \lim_{h\to 0} \frac{3(1+h)-\cancel{7}-3(1)+\cancel{7}}{h} = \lim_{h\to 0} \frac{\cancel{3}+3h-\cancel{3}}{h} = \boxed{3} \text{ ft/sec}$$

25. $f(t) = 1+\frac{1}{t},\ t=2$

$$\lim_{h\to 0} \frac{f(2+h)-f(2)}{h} = \lim_{h\to 0} \frac{\cancel{1}+\frac{1}{2+h}-\cancel{1}-\frac{1}{2}}{h} = \lim_{h\to 0} \frac{\frac{\cancel{2}-(\cancel{2}+h)}{2(2+h)}}{h} = \lim_{h\to 0} \frac{\frac{-\cancel{h}}{4+2h}}{\cancel{h}}$$

$$\lim_{h\to 0} \frac{-1}{4+2h} = \frac{-1}{4+0} = \boxed{\frac{-1}{4} \text{ ft/sec}}$$

27. $f(t) = 100 - 4.9t^2,\ t=2$

$$\lim_{h\to 0} \frac{f(2+h)-f(2)}{h} = \lim_{h\to 0} \frac{\cancel{100} - 4.9(2+h)^2 - \cancel{100} + 4.9(2)^2}{h} = \lim_{h\to 0} \frac{-4.9(\cancel{4}+4h+h^2) + 4.9\cancel{(4)}}{h}$$

$$\lim_{h\to 0} \frac{-19.6h - 4.9h^2}{h} = \lim_{h\to 0} -19.6 - 4.9h = -19.6 - 0 = \boxed{-19.6 \text{ m/s}}$$

(19.6 m/s downward)

29. $A(r) = \pi r^2,\ r=3$

$$\lim_{h\to 0} \frac{f(3+h)-f(3)}{h} = \lim_{h\to 0} \frac{\pi \cdot (3+h)^2 - \pi \cdot 3^2}{h} = \lim_{h\to 0} \frac{\pi(\cancel{9}+6h+h^2) - \cancel{\pi \cdot 9}}{h} = \lim_{h\to 0} \pi(6+h)$$

$$\pi(6+0) = \boxed{6\pi \text{ in}^2/\text{in}}$$

Area Radius

33. $f(x) = x^2 + 4x - 1,\ m = 0$

$$\lim_{h\to 0} \frac{f(a+h)-f(a)}{h} = \lim_{h\to 0} \frac{(a+h)^2 + 4(a+h) \cancel{-1} - a^2 - 4a \cancel{+1}}{h} = \lim_{h\to 0} \frac{\cancel{a^2} + 2ah + h^2 + \cancel{4a} + 4h - \cancel{a^2} - \cancel{4a}}{h}$$

$$\lim_{h\to 0} 2a + h + 4 = 2a + 0 + 4 = 2a + 4 \rightarrow 2a+4=0 \text{ when } a=-2$$

$$f(-2) = (-2)^2 + 4(-2) - 1 = 4 - 8 - 1 = -5 \rightarrow \boxed{(-2,-5)}$$

34. $f(x) = 3 - 4x - x^2,\ m = 0$

$$\lim_{h\to 0} \frac{f(a+h)-f(a)}{h} = \lim_{h\to 0} \frac{\cancel{3} - 4(a+h) - (a+h)^2 - \cancel{3} + 4a + a^2}{h} = \lim_{h\to 0} \frac{\cancel{-4a} - 4h - \cancel{a^2} - 2ah - h^2 + \cancel{4a} + \cancel{a^2}}{h}$$

$$\lim_{h\to 0} -4 - 2a - h = -4 - 2a - 0 = -4 - 2a \rightarrow -4-2a=0 \text{ when } a=-2$$

$$f(-2) = 3 - 4(-2) - (-2)^2 = 3 + 8 - 4 = 7 \rightarrow \boxed{(-2,7)}$$

35. $y = \frac{1}{x-1}$, $m=-1$

a) $\lim_{h\to 0} \frac{f(a+h)-f(a)}{h} = \lim_{h\to 0} \frac{\frac{1}{a+h-1} - \frac{1}{a-1}}{h} = \lim_{h\to 0} \frac{\frac{\cancel{a}-\cancel{1}-(\cancel{a}+h-\cancel{1})}{(a+h-1)(a-1)}}{h} = \lim_{h\to 0} \frac{\frac{-\cancel{h}}{(a+h-1)(a-1)}}{\cancel{h}}$

$\lim_{h\to 0} \frac{-1}{(a+h-1)(a-1)} = \frac{-1}{(a+0-1)(a-1)} = \frac{-1}{(a-1)^2}$

$\frac{-1}{(a-1)^2} = -1$ when $(a-1)^2 = 1 \to a-1 = \pm 1$ ↗ $a-1=1 \to a=2$ ↘ $a-1=-1 \to a=0$

$y(2) = \frac{1}{2-1} = \frac{1}{1} = 1 \to (2,1) \to y-1 = -1(x-2) \to y-1 = -x+2 \to \boxed{y=-x+3}$

$y(0) = \frac{1}{0-1} = \frac{1}{-1} = -1 \to (0,-1) \to y+1 = -1(x-0) \to y+1 = -x \to \boxed{y=-x-1}$

b) Normal = perpendicular to tangent

$m=1$, $(2,1) \to y-1 = 1(x-2) \to y-1 = x-2 \to \boxed{y=x-1}$

$m=1$, $(0,-1) \to \boxed{y=x-1}$

36. $y = 9-x^2$

$\lim_{h\to 0} \frac{f(a+h)-f(a)}{h} = \lim_{h\to 0} \frac{\cancel{9}-(a+h)^2-\cancel{9}+a^2}{h} = \lim_{h\to 0} \frac{\cancel{-a^2}-2ah-h^2+\cancel{a^2}}{h} = \lim_{h\to 0} -2a-h = -2a-0 = -2a$ slope

$(1,12)$ $(x, 9-x^2)$

$\frac{\Delta y}{\Delta x} = \frac{9-a^2-12}{a-1} = -2a \to \frac{-3-a^2}{a-1} = -2a \to -3-a^2 = -2a(a-1)$

$-3-a^2 = -2a^2+2a$

$a^2-2a-3=0$

$(a-3)(a+1)=0 \to a=3, a=-1$

$y(3) = 9-3^2 = 0 \to (3,0)$, $m=-2(3)=-6$

$y(-1) = 9-(-1)^2 = 8 \to (-1,8)$, $m=-2(-1)=2$

$y-0 = -6(x-3) \to \boxed{y=-6x+18}$

$y-8 = 2(x+1) \to y-8 = 2x+2 \to \boxed{y=2x+10}$

37.

Year	x	Export	
2000	0	844	
(Q_1) 2004	4	381	(Q_1)
(Q_2) 2005	5	313	(Q_2)
2006	6	281	
(Q_3) 2007	7	448	(Q_3)
(P) 2008	8	389	(P)

$$PQ_1 = \frac{389-381}{8-4} = 2$$

$$PQ_2 = \frac{389-313}{8-5} = 25.333$$

$$PQ_3 = \frac{389-448}{8-7} = -59$$