Derivative of a Function (Section 3.1)

* Derivative: Newton's Quotient of a "derived" function

SLOPE! Written as $f'(x) \rightarrow$ "f prime of x"

$$\boxed{f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}}$$

*Provided that the limit exists!

ex: $f(x) = x^3$ Find the derivative.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{(x+h)^3 - x^3}{h} = \lim_{h \to 0} \frac{\cancel{x^3} + 3x^2h + 3xh^2 + h^3 - \cancel{x^3}}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h}(3x^2 + 3xh + h^2)}{\cancel{h}} \boxed{= 3x^2}$$

* Alternate Definition of a Derivative (usually used @ a point):

$$\boxed{f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}}$$

← This should remind you of $\frac{y_2 - y_1}{x_2 - x_1} = m$

ex: $f(x) = \sqrt{x}$ Find the derivative @ $x = 4$.

$$f'(x) = \lim_{x \to 4} \frac{f(x) - f(4)}{x - 4} = \lim_{x \to 4} \left(\frac{\sqrt{x} - 2}{x - 4} \right) = \lim_{x \to 4} \frac{\cancel{(\sqrt{x} - 2)}}{(\sqrt{x} + 2)\cancel{(\sqrt{x} - 2)}} = \lim_{x \to 4} \left(\frac{1}{\sqrt{x} + 2} \right)$$

$$= \frac{1}{\sqrt{4} + 2} = \boxed{\frac{1}{4}}$$

*Notations of a Derivative:

$f'(x)$: "f prime of x"

y': "y prime"

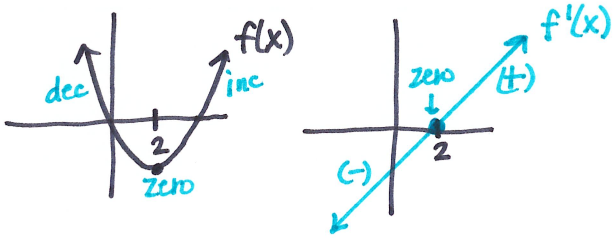
$\frac{d}{dx} f(x)$: "the derivative of f(x)"

$\frac{dy}{dx}$: "the derivative of y w/respect to x" OR "dy dx"

* Relationship between $f(x)$ & $f'(x)$:

1. when $f(x)$ is increasing → $f'(x)$ is positive
2. when $f(x)$ is decreasing → $f'(x)$ is negative
3. Any max/min of $f(x)$ → $f'(x) = 0$

ex:

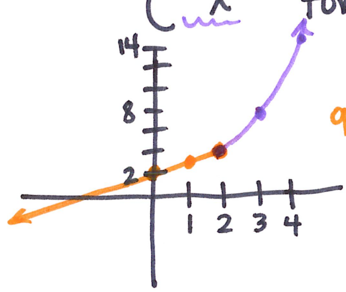


* One-sided Derivatives: If $f(x)$ is differentiable on a closed interval $[a,b]$

then: $\lim\limits_{h \to 0^+} \dfrac{f(a+h)-f(a)}{h}$: Right-hand derivative @ $x=a$

$\lim\limits_{h \to 0^-} \dfrac{f(a+h)-f(a)}{h}$: Left-hand derivative @ $x=a$

ex: $y = \begin{cases} x+2 & \text{for } x<2 \\ x^2 & \text{for } x \geq 2 \end{cases}$ Find the left & right-hand derivatives @ $x=2$ using one-sided limits.



x	y
open* 2	4
1	3
0	2
⋮	

x	y
2	4
3	9
4	16

since these are equal... f(x) is continuous!

Left: $\lim\limits_{h \to 0^-} \dfrac{f(2+h)-f(2)}{h} = \lim\limits_{h \to 0^-} \dfrac{[(\cancel{2}+h)+\cancel{2}]-\cancel{4}}{h} = \lim\limits_{h \to 0^-} \dfrac{h}{h} \boxed{=1}$

Right: $\lim\limits_{h \to 0^+} \dfrac{f(2+h)-f(2)}{h} = \lim\limits_{h \to 0^+} \dfrac{(2+h)^2-4}{h} = \lim\limits_{h \to 0^+} \dfrac{\cancel{4}+4h+h^2-\cancel{4}}{h} = \lim\limits_{h \to 0^+} \dfrac{\cancel{h}(4+h)}{\cancel{h}} \boxed{=4}$