Section 3.2: 1-15 odd, 31-35 odd, 38

1. $y = x^2$, $y = x$

$=x^2$ $\lim_{h\to 0} \frac{f(0+h)-f(0)}{h} = \lim_{h\to 0} \frac{h^2 - 0}{h} = \lim_{h\to 0} h = 0$ as $x \to 0^-$

$=x$ $\lim_{h\to 0} \frac{f(0+h)-f(0)}{h} = \lim_{h\to 0} \frac{h-0}{h} = 1$ as $x \to 0^+$

$0 \neq 1$, so not differentiable at $x = 0$.

3. $y = \sqrt{x}$, $y = 2x - 1$

$y=\sqrt{x}$ $\lim_{h\to 0} \frac{f(1+h)-f(1)}{h} = \lim_{h\to 0} \frac{\sqrt{1+h}-1}{h} \frac{(\sqrt{1+h}+1)}{(\sqrt{1+h}+1)} = \lim_{h\to 0} \frac{1+h-1}{h(\sqrt{1+h}+1)} = \lim_{h\to 0} \frac{1}{\sqrt{1+h}+1}$

$\frac{1}{\sqrt{1+0}+1} = \frac{1}{\sqrt{1}+1} = \frac{1}{1+1} = \frac{1}{2}$ as $x \to 1^-$

$y=2x-1$ $\lim_{h\to 0} \frac{f(1+h)-f(1)}{h} \lim_{h\to 0} \frac{2(1+h)-1-2(1)+1}{h} = \lim_{h\to 0} \frac{2+2h-2}{h} = 2$ as $x \to 1^+$

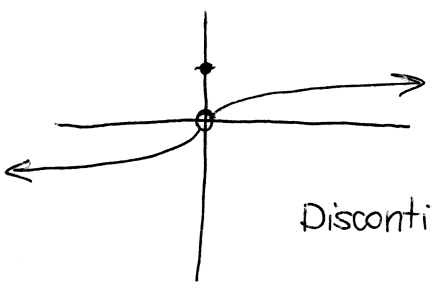
$\frac{1}{2} \neq 2$, so not differentiable at $x = 1$.

5. a) $(-3, 2)$
   b) $x = -3$, $x = 2$
   c) None

7. a) $(-3, 0) \cup (0, 3)$
   b) $x = -3$, $x = 3$
   c) $x = 0$

9. a) $(-1, 0) \cup (0, 2)$
   b) $x = -1$, $x = 0$, $x = 2$
   c) None

11. $y = \begin{cases} \tan^{-1} x, & x \neq 0 \\ 1, & x = 0 \end{cases}$



13. $y = x + \sqrt{x^2 + 2}$

Graph on calc → Corner

15. $y = 3x - 2|x| - 1$

Graph on calc → Corner

31. $f(x) = \dfrac{x^3 - 8}{x^2 - 4x - 5} = \dfrac{(x-2)(x^2+2x+4)}{(x-5)(x+1)}$

VA at $x = 5$, $x = -1$, so infinite discontinuity & not differentiable.

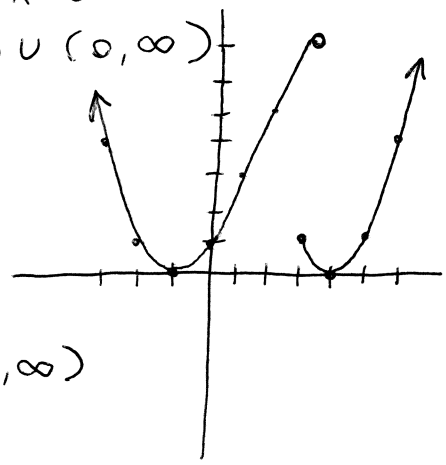
Differentiable: $(-\infty, -1) \cup (-1, 5) \cup (5, \infty)$

33. $P(x) = \sin(|x|) - 1$

Graph on calc → cusp at $x = 0$

Differentiable: $(-\infty, 0) \cup (0, \infty)$

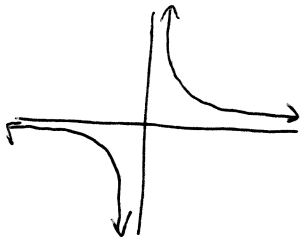
35. $g(x) = \begin{cases} (x+1)^2, & x \le 0 \\ 2x+1, & 0 < x < 3 \\ (4-x)^2, & x \ge 3 \end{cases}$



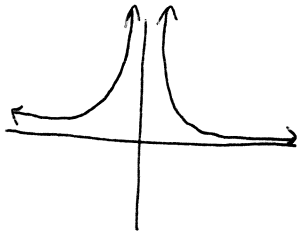
Jump discontinuity at $x = 3$

Differentiable: $(-\infty, 3) \cup (3, \infty)$

38. $\frac{1}{x}$



$\frac{1}{x^2}$

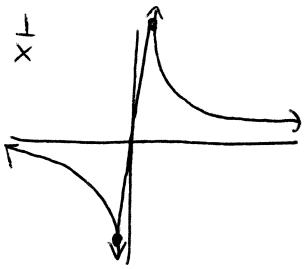


a) Not differentiable bc not continuous at $x = 0$.

b) nDeriv $(1/x, x, 0) = 1{,}000{,}000$

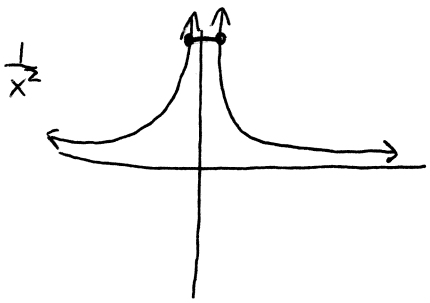
nDeriv $(1/x^2, x, 0) = 0$

c) $\frac{1}{x}$



Odd, steep to connect bc opposite y values

$\frac{1}{x^2}$



Even, flat to connect bc same y values

$$\frac{f(0+0.001) - f(0-0.001)}{0.002}$$