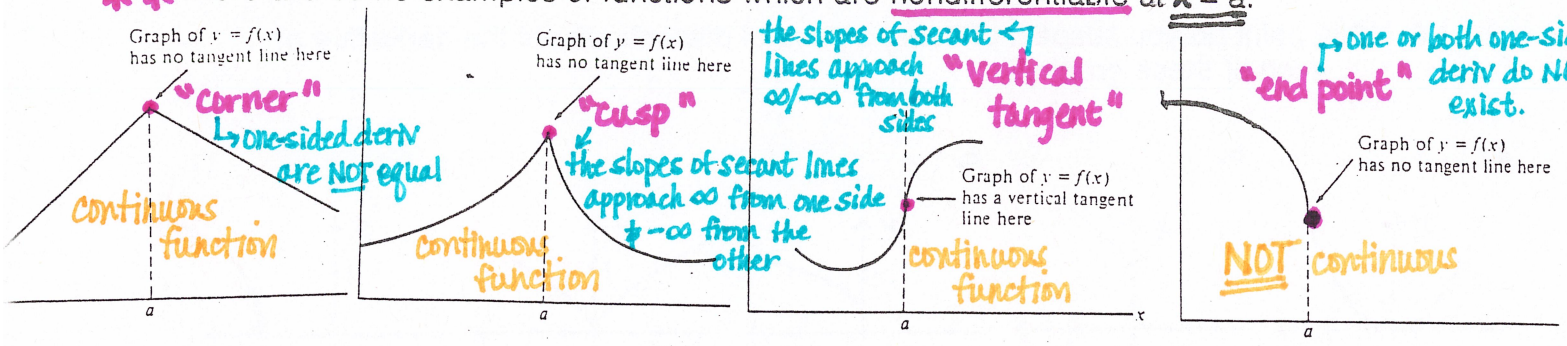
NOTES Section 3.2

DIFFERENTIABILITY AND CONTINUITY

A function is differentiable at a point if you can take the derivative at that point. That means you can draw one unique tangent which has a slope at that point.

** Here are some examples of functions which are nondifferentiable at x = a.

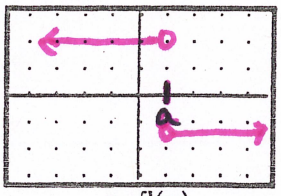


a. b. c. d.

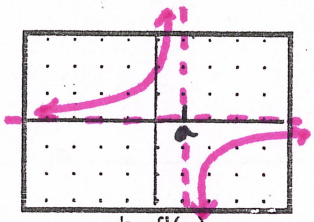
1. How does that relate to continuity? Which of the functions above are continuous at x = a?

$\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x) = f(a)$

Sketch the derivative of functions a and b.

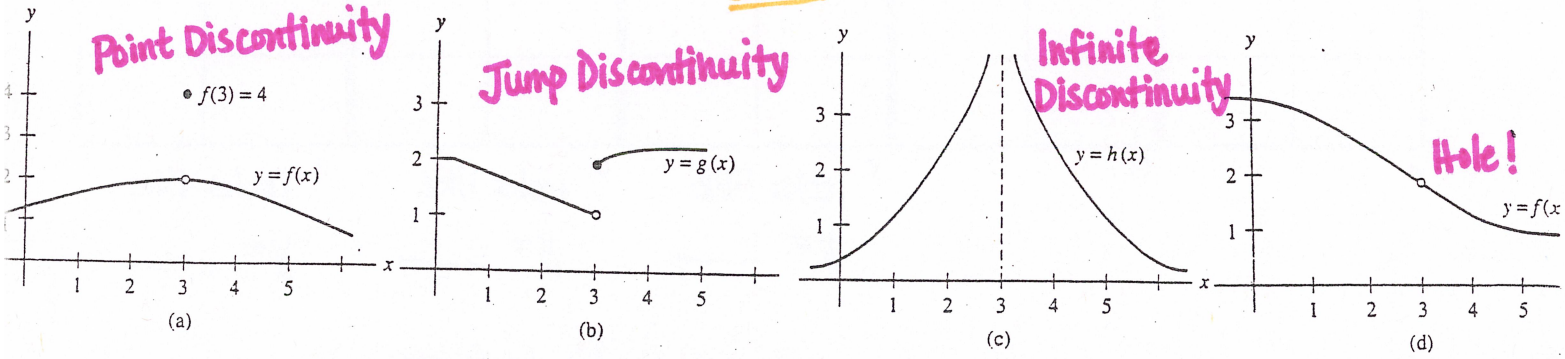


a. f'(x)



b. f'(x)

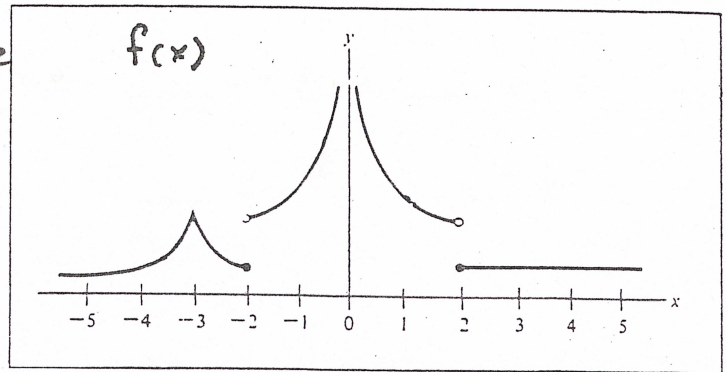
2. Which of the functions below are continuous at x = 3? NONE!
Which are differentiable at x = 3? NONE



NC if it is not continuous

3. For f(x) write C if it is continuous and
D if it is differentaible
ND if it is not differentiable
for these values of x:

a. x = 0 — NC, ND
b. x = .001 — C, D
c. x = -3 — C, ND → cusp
d. x = 3 — C, D
e. x = -2 — NC, ND
f. x = 2 — NC, ND



4. Is it possible for a function to be continuous at a point and not differentiable? yes!

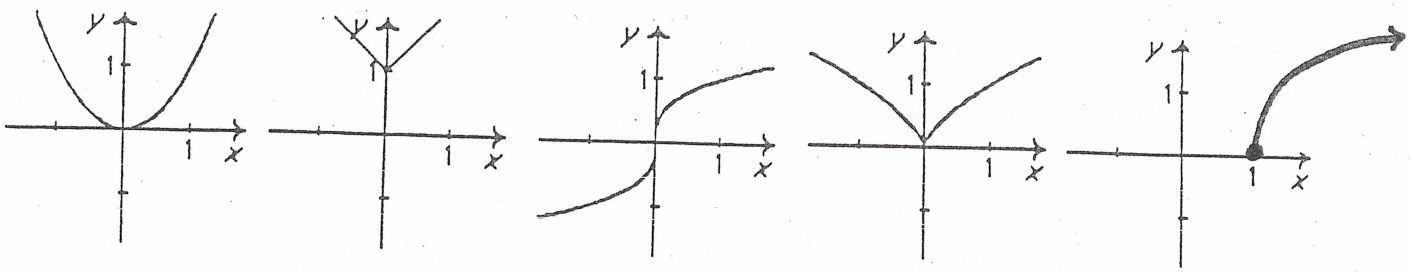
** Is is possible for a function to be differentiable at a point and not be continuous? No

5. What is the relationship between continuous and differentiable?

If a function, f(x), is differentiable @ x=a, then the function must be continuous @ x=a.

6. What point do you suspect of being a point of discontinuity of the derivative of each of these graphs?

a. b. c. d. e.



$f(x) = x^2$ $f(x) = |x| + 1$ $f(x) = x^{1/3}$ $f(x) = x^{2/3}$ $f(x) = \sqrt{x-1}$

x-coordinates of points of discontinuity of

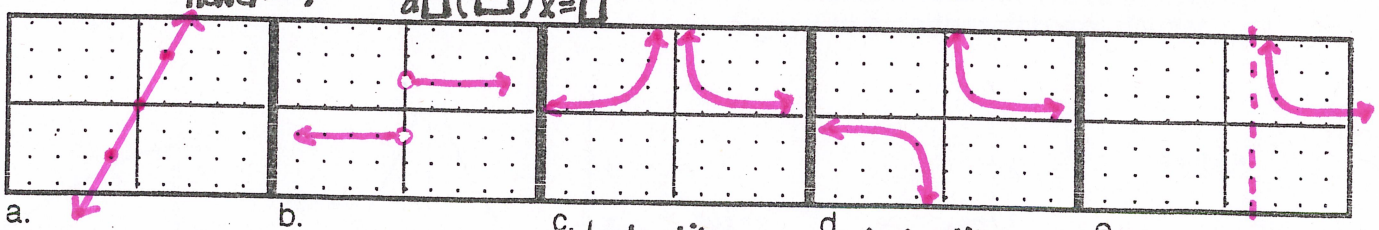
a. None b. x=0 c. x=0 d. x=0 e. x=1

7. Use your calculator to sketch the derivative of each and see if you were correct.

Y_1 = your function Y_2 = nDeriv(Y_1,x,x) (found in MATH 8)

older → $Y_2 = \frac{d}{d\square}(\square)_{x=\square}$

newer →



a. b. c. looks like $y = \frac{1}{x^2}$ d. looks like $y = \frac{1}{x}$ e. looks like $y = \frac{1}{\sqrt{x-1}}$

8. Find the derivative of each of the functions in problem 6 and see if the mathematics supports your findings. What is the domain of the derivative of each function?

a. $y' = 2x$ b. $y' = \frac{|x|}{x}$ c. $y' = \frac{1}{3}x^{-2/3}$ d. $y' = \frac{2}{3}x^{-1/3}$ e. $y' = \frac{1}{2}(x-1)^{-1/2}$

D: $(-\infty, \infty)$ D: $(-\infty, 0) \cup (0, \infty)$ D: $(-\infty, 0) \cup (0, \infty)$ D: $(-\infty, 0) \cup (0, \infty)$ D: $(1, \infty)$