# Section 3.3: 1-41 odd, 42, 46-52 all

1. $y = -x^2 + 3$

$\frac{dy}{dx} = \boxed{-2x}$

3. $y = 2x + 1$

$\frac{dy}{dx} = \boxed{2}$

5. $y = \frac{1}{3}x^3 + \frac{1}{2}x^2 + x$

$\frac{dy}{dx} = \boxed{x^2 + x + 1}$

7. $y = x^3 - 2x^2 + x + 1$

$y' = 3x^2 - 4x + 1 = 0$ x to 3, x to -4 → -3 & -1

$3x^2 - 3x - x + 1 = 0$

$3x(x-1) - 1(x-1) = 0$

$(x-1)(3x-1) = 0$

$\boxed{x = 1,\ x = 1/3}$

9. $y = x^4 - 4x^2 + 1$

$y' = 4x^3 - 8x = 0$

$4x(x^2 - 2) = 0$

$\boxed{x = 0,\ x = \pm\sqrt{2}}$

11. $y = 5x^3 - 3x^5$

$y' = 15x^2 - 15x^4 = 0$

$15x^2(1 - x^2) = 0$

$\boxed{x = 0,\ x = \pm 1}$

13. $y = (x+1)(x^2+1)$

a) $y' = (x+1)(2x) + (x^2+1)(1) = 2x^2 + 2x + x^2 + 1 = \boxed{3x^2 + 2x + 1}$

b) $y = x^3 + x^2 + x + 1$

$y' = \boxed{3x^2 + 2x + 1}$

15. $y = (x^3+x+1)(x^4+x^2+1)$

$\frac{dy}{dx} = (x^3+x+1)(4x^3+2x) + (x^4+x^2+1)(3x^2+1)$

$\frac{dy}{dx} = 4x^6+4x^4+4x^3+2x^4+2x^2+2x+3x^6+3x^4+3x^2+x^4+x^2+1$

$\frac{dy}{dx} = \boxed{7x^6+10x^4+4x^3+6x^2+2x+1}$

17. $y = \frac{2x+5}{3x-2}$

$\frac{dy}{dx} = \frac{(3x-2)(2)-(2x+5)(3)}{(3x-2)^2} = \frac{6x-4-6x-15}{(3x-2)^2} = \boxed{\frac{-19}{(3x-2)^2}}$

19. $y = \frac{(x-1)(x^2+x+1)}{x^3} = \frac{x^3+x^2+x-x^2-x-1}{x^3} = \frac{x^3-1}{x^3} = \frac{x^3}{x^3} - \frac{1}{x^3} = 1-x^{-3}$

$\frac{dy}{dx} = 3x^{-4} = \boxed{\frac{3}{x^4}}$

21. $y = \frac{x^2}{1-x^3}$

$\frac{dy}{dx} = \frac{(1-x^3)(2x)-(x^2)(-3x^2)}{(1-x^3)^2} = \frac{2x-2x^4+3x^4}{(1-x^3)^2} = \boxed{\frac{2x+x^4}{(1-x^3)^2}}$

23. $u(0)=5,\ u'(0)=-3,\ v(0)=-1,\ v'(0)=2$

a) $\frac{d}{dx}(vu) = v\cdot u' + u\cdot v' = (-1)(-3)+(5)(2) = 3+10 = \boxed{13}$

b) $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\cdot u' - u\cdot v'}{v^2} = \frac{(-1)(-3)-(5)(2)}{(-1)^2} = \frac{3-10}{1} = \boxed{-7}$

c) $\frac{d}{dx}\left(\frac{v}{u}\right) = \frac{u\cdot v' - v\cdot u'}{u^2} = \frac{(5)(2)-(-1)(-3)}{5^2} = \frac{10-3}{25} = \boxed{\frac{7}{25}}$

d) $\frac{d}{dx}(7v-2u) = 7v'-2u' = 7(2)-2(-3) = 14+6 = \boxed{20}$

25. $y = x^2+5x$

$y' = 2x+5$

$y'(3) = 2(3)+5 = \boxed{11 \rightarrow \text{iii}}$

27. $y = \frac{x^3+1}{2x} = \frac{x^3}{2x} + \frac{1}{2x} = \frac{1}{2}x^2 + \frac{1}{2}x^{-1}$

$y' = x - \frac{1}{2}x^{-2} = x - \frac{1}{2x^2}$

$y'(1) = 1 - \frac{1}{2\cdot 1^2} = 1 - \frac{1}{2} = \frac{1}{2}$ slope

$y(1) = \frac{1^3+1}{2(1)} = \frac{2}{2} = 1 \rightarrow (1,1)$

$y - 1 = \frac{1}{2}(x-1)$

$y - 1 = \frac{1}{2}x - \frac{1}{2}$

$\boxed{y = \frac{1}{2}x + \frac{1}{2}}$

29. $y = 4x^{-2} - 8x + 1$

$\frac{dy}{dx} = -8x^{-3} - 8 = \boxed{\frac{-8}{x^3} - 8}$

31. $y = \frac{\sqrt{x}-1}{\sqrt{x}+1}$

$\frac{dy}{dx} = \frac{(\sqrt{x}+1)\left(\frac{1}{2\sqrt{x}}\right) - (\sqrt{x}-1)\left(\frac{1}{2\sqrt{x}}\right)}{(\sqrt{x}+1)^2} = \frac{\frac{1}{2\sqrt{x}}\left[\sqrt{x}+1-\sqrt{x}+1\right]}{(\sqrt{x}+1)^2} = \frac{\frac{1}{2\sqrt{x}}\cdot\frac{2}{1}}{(\sqrt{x}+1)^2}$

$\frac{dy}{dx} = \frac{\frac{1}{\sqrt{x}}}{(\sqrt{x}+1)^2} = \frac{1}{\sqrt{x}}\cdot\frac{1}{(\sqrt{x}+1)^2} = \boxed{\frac{1}{\sqrt{x}(\sqrt{x}+1)^2}}$

33. $y = x^4 + x^3 - 2x^2 + x - 5$

$y' = 4x^3 + 3x^2 - 4x + 1$

$y'' = 12x^2 + 6x - 4$

$y''' = 24x + 6$

$y'''' = 24$

35. $y = x^{-1} + x^2$

$y' = -x^{-2} + 2x$

$y'' = 2x^{-3} + 2$

$y''' = -6x^{-4}$

$y'''' = 24x^{-5}$

37. $y = x^3 - 3x + 1$ $(2,3)$

$y' = 3x^2 - 3$

$y'(2) = 3 \cdot 2^2 - 3 = 9$

$\perp$ slope $= -1/9$

$y - 3 = -\frac{1}{9}(x-2)$

$y - 3 = -\frac{1}{9}x + \frac{2}{9}$

$\boxed{y = -\frac{1}{9}x + \frac{29}{9}}$

39. $y = 2x^3 - 3x^2 - 12x + 20$, $m = 0$

$y' = 6x^2 - 6x - 12 = 0$

$x^2 - x - 2 = 0$

$(x-2)(x+1) = 0$

$x = 2, \; x = -1$

$y(2) = 2 \cdot 2^3 - 3 \cdot 2^2 - 12 \cdot 2 + 20$

$y(2) = 16 - 12 - 24 + 20 = 0 \rightarrow \boxed{(2,0)}$

$y(-1) = 2(-1)^3 - 3(-1)^2 - 12(-1) + 20$

$y(-1) = -2 - 3 + 12 + 20 = 27 \rightarrow \boxed{(-1,27)}$

41. $y = \frac{4x}{x^2+1}$ at $(0,0)$ and $(1,2)$

$y' = \frac{(x^2+1)(4) - (4x)(2x)}{(x^2+1)^2} = \frac{4x^2 + 4 - 8x^2}{(x^2+1)^2} = \frac{-4x^2+4}{(x^2+1)^2}$

$y'(0) = \frac{0+4}{(0+1)^2} = \frac{4}{1} = 4$ at $(0,0) \rightarrow$ $y - 0 = 4(x-0) \rightarrow \boxed{y = 4x}$

$y'(1) = \frac{-4+4}{(1+1)^2} = \frac{0}{4} = 0$ at $(1,2) \rightarrow$ $y - 2 = 0(x-1) \rightarrow \boxed{y = 2}$

42. $y = \frac{8}{4+x^2}$ at $(2,1)$

$y' = \frac{(4+x^2)(0) - (8)(2x)}{(4+x^2)^2} = \frac{-16x}{(4+x^2)^2}$

$y'(2) = \frac{-32}{(4+4)^2} = \frac{-32}{8^2} = \frac{-32}{64} = -\frac{1}{2}$ at $(2,1)$

$y - 1 = -\frac{1}{2}(x-2)$

$y - 1 = -\frac{1}{2}x + 1$

$\boxed{y = -\frac{1}{2}x + 2}$

46. $P = \frac{nRT}{V-nb} - \frac{an^2}{V^2} = nRT(V-nb)^{-1} - an^2V^{-2}$

$\frac{dP}{dV} = -nRT(V-nb)^{-2} + 2an^2V^{-3} = \boxed{\frac{-nRT}{(V-nb)^2} + \frac{2an^2}{V^3}}$

47. $s = 4.9t^2$

$\frac{ds}{dt} = 9.8t$

$\frac{d^2s}{dt^2} = 9.8$

48. $R = M^2\left(\frac{c}{2} - \frac{M}{3}\right) = \frac{c}{2}M^2 - \frac{1}{3}M^3$

$\frac{dR}{dM} = cM - M^2$

49. $A = \pi r^2$

$\frac{dA}{dr} = 2\pi r = C$

Adding another ring around the outside of a circle (circumference) expands its area.

50. $V = \frac{4}{3}\pi r^3$

$\frac{dV}{dr} = 4\pi r^2 = SA$

Adding another shell around the outside of a sphere (surface area) expands its volume.

51. Production = Trees × Yield

(Production)' = Trees × (Yield)' + Yield × (Trees)' = $156 \cdot 1.5 + 12 \cdot 13$

Rate of change of production = $234 + 156 =$ **390 bushels per year**

52. $\text{Fee} = \frac{\text{Cost}}{\text{Members}}$

$(\text{Fee})' = \frac{(\text{Members}) \times (\text{Cost})' - (\text{Cost}) \times (\text{Members})'}{(\text{Members})^2} = \frac{65 \cdot 10 - 250 \cdot 6}{65^2}$

Rate of change of fee $= \frac{650 - 1500}{4225} = \frac{-850}{4225} =$ **−0.201 dollars per year**

Fee per member is decreasing at a rate of ~20¢ per year.