Rules of Differentials (Section 3.3)

* Derivative of a constant: If $f(x)=c$, then $f'(x)=0$

* Derivative of a power function: If $f(x)=x^n$, then $f'(x)=nx^{n-1}$

* Sum/Difference Rule: If $f(x)=u+v$, then $f'(x)=u'+v'$

* Second derivatives: If $f(x)=u$, then $f'(x)$ is the 1st derivative & $f''(x)$ is the 2nd derivative.

ex: $f(x)=3+x^2+x^1-x^4$

$f'(x)=0+2x^1+1x^0-4x^3$

$\boxed{f'(x)=-4x^3+2x+1}$

ex: $f(x)=4x^2-6x+\sqrt{x}$ → $x^{1/2}$

$f'(x)=4\cdot 2x^1-6\cdot 1x^0+\frac{1}{2}x^{-1/2}$

$\boxed{f'(x)=8x-6+\frac{1}{2}x^{-1/2}}$

ex: $f(x)=\frac{x^3}{4}+\frac{x^2}{2}-x+7$

$f'(x)=\frac{1}{4}\cdot 3x^2+\frac{1}{2}\cdot 2x-1+0$

$\boxed{f'(x)=\frac{3}{4}x^2+x-1}$

* Product Rule: $\frac{d}{dx}(u\cdot v)=u\cdot\frac{dv}{dx}+v\cdot\frac{du}{dx}$

"1st times derv. of 2nd plus 2nd times derv of the 1st"

ex: $f(x)=(x+3)(x^3-7)$

$f'(x)=(x+3)(3x^2)+(x^3-7)(1)$

$=3x^3+9x^2+x^3-7$

$\boxed{=4x^3+9x^2-7}$

ex: $f(x)=(x^2-2)(x^{-1}+1)$ OR FOIL first!

$f(x)=x^1-2x^{-1}+x^2-2$

$f'(x)=1+2x^{-2}+2x$

Same

$f'(x)=(x^2-2)(-x^{-2})+(x^{-1}+1)(2x)$

$=-1+2x^{-2}+2+2x$

$=1+2x+\frac{2}{x^2}$

* Quotient Rule: $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\cdot\frac{du}{dx} - u\cdot\frac{dv}{dx}}{v^2}$ "Ho d hi minus Hi d ho over Ho Ho!"

ex: $y = \frac{2x+7}{x^2-3}$ $y' = \frac{(x^2-3)(2)-(2x+7)(2x)}{(x^2-3)^2} = \frac{2x-6-4x^2-14x}{(x^2-3)^2}$

$= \frac{-2x^2-14x-6}{(x^2-3)^2}$

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ex: $y = \frac{1-x}{x^2+1}$ $y' = \frac{(x^2+1)(-1)-(1-x)(2x)}{(x^2+1)^2} = \frac{-x^2-1-2x+2x^2}{(x^2+1)^2}$

$= \frac{x^2-2x-1}{(x^2+1)^2}$

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ex: $\frac{d}{dx}(u\cdot v) = u(0)\cdot v'(0) + v(0)\cdot u'(0)$ @ $x=0$

$= (5\cdot 2) + (-1\cdot -3) = 10+3 = 13$

#37 $y = x^3-3x+1$ @ $(2,3)$

$y' = 3x^2-3$

$y'(2) = 3(2)^2-3 = 9$ ← slope!

$\perp m = -\frac{1}{9}$ $y-3 = -\frac{1}{9}(x-2)$ ← Could stop here!

$y = -\frac{1}{9}x + \frac{2}{9} + 3$

$y = -\frac{1}{9}x + \frac{29}{9}$

#46 $P = \frac{nRT}{V-nb} - \frac{an^2}{V^2}$ → an^2V^{-2}

$\frac{dP}{dV} = \frac{(V-nb)(0)-(nRT)(1)}{(V-nb)^2} - (-2an^2V^{-3})$

$\frac{dP}{dV} = \frac{-nRT}{(V-nb)^2} + \frac{2an^2}{V^3}$

#49 $\Delta A = C\cdot$ width of r

$\Delta A = (2\pi r)(\Delta r)$

$\frac{\Delta A}{\Delta r} = 2\pi r$

#51 $t(x)$ = # of trees $\quad t(0)=156 \quad y(0)=12$

$y(x)$ = yield per tree $\quad t'(0)=13 \quad y'(0)=1.5$

$\text{yield} = t(x)\cdot y(x)$

$\text{yield}' = t(x)\cdot y'(x) + y(x)\cdot t'(x)$ @ $x=0$

$= t(0)\cdot y'(0) + y(0)\cdot t'(0)$

$= (156\cdot 1.5) + (12\cdot 13)$

$= 390$ bushels/yr