Section 3.4: 1-15 odd, 8, 18, 24, 28-30, 32-35, 38

1. a) $V = s^3$

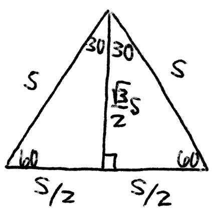
   b) $\frac{dV}{ds} = 3s^2$

   c) $3(1)^2 = 3$

   $3(5)^2 = 75$

   d) in^3 per in

3. a)



$A = \frac{1}{2} \cdot s \cdot \frac{\sqrt{3}}{2} s$

$A = \frac{\sqrt{3}}{4} s^2$

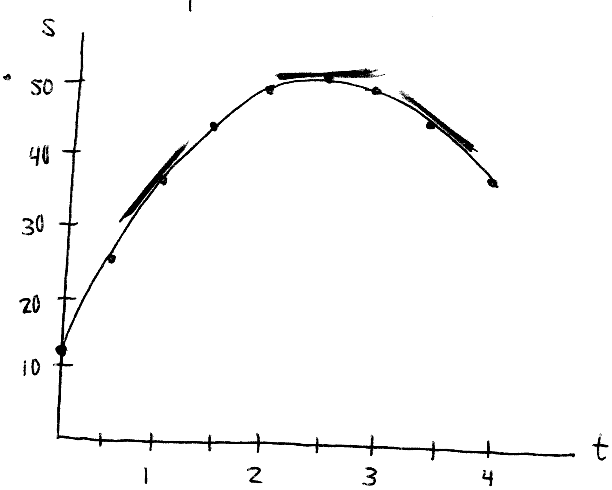
   b) $\frac{dA}{ds} = \frac{\sqrt{3}}{2} s$

   c) $\frac{\sqrt{3}}{2} \cdot 2 = \sqrt{3}$

   $\frac{\sqrt{3}}{2} \cdot 10 = 5\sqrt{3}$

   d) in^2 per in

5.



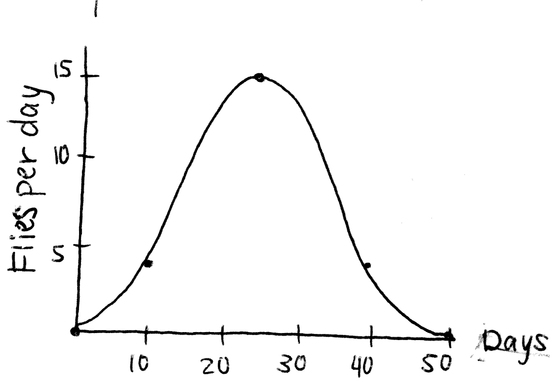
b) $t = 1.0: \frac{\Delta y}{\Delta x} = \frac{44-26}{1.5-0.5} = \frac{18}{1} = \boxed{18}$

$t = 2.5: \frac{\Delta y}{\Delta x} = \frac{48.5-48.5}{3.0-2.0} = \frac{0}{1} = \boxed{0}$

$t = 3.5: \frac{\Delta y}{\Delta x} = \frac{36.5-48.5}{4.0-3.0} = \frac{-12}{1} = \boxed{-12}$

7.

a) p'



b) Fastest: ~ Day 25

Slowest: Day 0 & Day 50

9. a) Forward = + velocity : $0<t<1$, $5<t<7$

Backward = − velocity: $1<t<5$

Speed up = velocity + & inc or − & dec : $1<t<2$ and $5<t<6$

Slow down = velocity + & dec or − & inc : $0<t<1$, $3<t<5$, $6<t<7$

b) Acceleration = slope of velocity

+ : $3<t<6$

− : $0<t<2$, $6<t<7$

0 : $2<t<3$, $7<t\le 9$

c) Greatest speed = max velocity, either + or −

$t=0$, $2<t<3$

d) Still = velocity 0

$t=1$, $t=5$, $7<t\le 9$

Instant

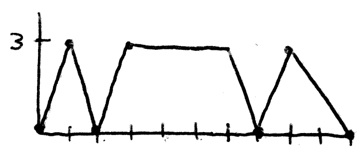
11. a) Change direction = velocity + to − or − to +

$t=2$, $t=7$

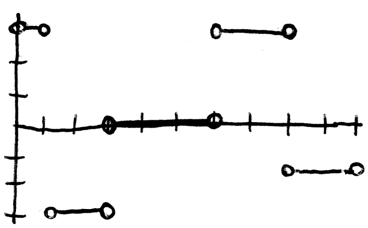
b) Constant speed = no change in velocity

$3<t<6$

c) Speed = |velocity|



d) Acceleration = slope of velocity



13. $s = 24t - 0.8t^2$

a) $v = s' = \boxed{24 - 1.6t \text{ m/s}}$

$a = v' = s'' = \boxed{-1.6 \text{ m/s}^2}$

b) Max height = velocity 0

$24 - 1.6t = 0 \rightarrow t = \boxed{15 \text{ s}}$

c) $s(15) = 24(15) - 0.8(15)^2 = \boxed{180 \text{ m}}$

d) $24t - 0.8t^2 = 90$

$-0.8t^2 + 24t - 90 = 0$

$\dfrac{-24 \pm \sqrt{24^2 - 4(-0.8)(-90)}}{2(-0.8)}$ $\rightarrow \boxed{4.393 \text{ s}}$, $\boxed{25.607 \text{ s}}$

e) $24t - 0.8t^2 = 0$

$t(24 - 0.8t) = 0$

$t = 0$, $24 - 0.8t = 0$

$24 = 0.8t$

$\boxed{t = 30 \text{ s}}$

15. $s = 24t - 4.9t^2$

$v = s' = 24 - 9.8t$

Max height = velocity 0

$24 - 9.8t = 0$

$t = 2.449$ s

$s(2.449) = 24(2.449) - 4.9(2.449)^2 = \boxed{29.388 \text{ m}}$

8. $Q(t) = 200(30-t)^2$

$\text{Average} = \frac{\Delta Q}{\Delta t} = \frac{Q(10) - Q(0)}{10 - 0} = \frac{-100{,}000 \text{ gal}}{10 \text{ min}} = \boxed{-10{,}000 \text{ gal/min}}$ (- bc water flows out)

$Q(10) = 200(30-10)^2 = 80{,}000$

$Q(0) = 200(30-0)^2 = 180{,}000$

$Q(t) = 200(900 - 60t + t^2) = 180{,}000 - 12{,}000t + 200t^2$

$Q'(t) = -12{,}000 + 400t$

$Q'(10) = -12{,}000 + 400(10) = \boxed{-8{,}000 \text{ gal/min}}$

18. a) $v(2) = 190$ ft/s

b) 2 s

c) Max height = velocity 0 → 8 s

d) ~11 s, $v(11) = -90$ ft/s (downward)

e) Velocity - & dec : $8 < t < 11$ → 3 s

f) Greatest slope of velocity : just before $t = 2$ s

Acceleration = slope of velocity

Constant (same slope) : $2 < t < 11$ (during free fall)

24. $v = 2t^3 - 9t^2 + 12t - 5$

$a = v' = 6t^2 - 18t + 12 = 0$

$t^2 - 3t + 2 = 0$

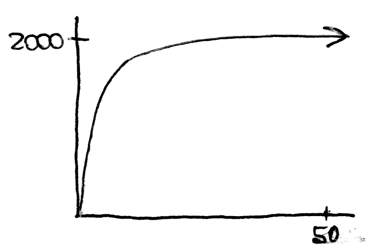
$(t-2)(t-1) = 0$

$t = 2, t = 1$

$v(2) = 16 - 36 + 24 - 5 = -1$ m/s → speed = |vel| → $\boxed{1 \text{ m/s}}$

$v(1) = 2 - 9 + 12 - 5 = \boxed{0 \text{ m/s}}$

28. a) $r(x) = 2000\left(1 - \frac{1}{x+1}\right)$



$x \geq 0$
Can't sell a negative number of desks

b) $r(x) = 2000 - 2000(x+1)^{-1}$

Marginal revenue $= r'(x)$

$r'(x) = 2000(x+1)^{-2} = \frac{2000}{(x+1)^2}$

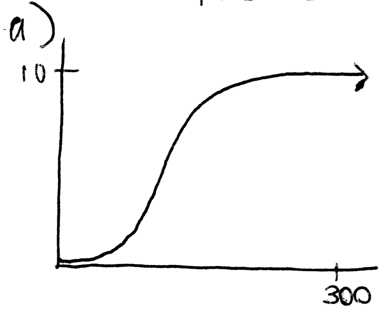
c) Rate of change at $5 = r'(5) = \frac{2000}{(5+1)^2} = \$55.56$

d) $\lim_{x \to \infty} \frac{2000}{(x+1)^2} = \frac{2000}{(\infty+1)^2} = \frac{2000}{\infty^2} = 0$

When a very large number of desks are sold, there is almost no change in marginal revenue.

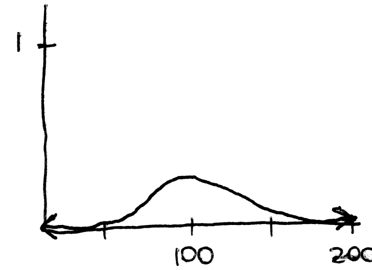
29. $P(x) = \frac{10}{1 + 50 \cdot 2^{5-0.1x}}$

a)



b) $x \geq 0$ (Can't sell a negative number of software packages)

c) $Y_2 = \text{nDeriv}(Y_1, x, x)$



Greatest slope around $x = 100$

d) Max $P'(x) = 0.173$ when $x = 106.439 \rightarrow$ use $x = 106$ units sold

$P(106) = \frac{10}{1 + 50 \cdot 2^{5 - 0.1(106)}} = 4.924$ thousand $= \$4,924$

e) $P'(50) = 0.0133$ thousand $= \$13.3$ / package

$P'(100) = 0.165$ thousand $= \$165$ / package

$P'(125) = 0.118$ thousand $= \$118$ / package

$P'(150) = 0.0308$ thousand $= \$30.8$ / package

$P'(175) = 0.00588$ thousand $= \$5.88$ / package

$P'(300) = 0$ thousand $= \$0$ / package

29. f) $\lim\limits_{x\to\infty} \dfrac{10}{1+\dfrac{50\cdot 2^5}{2^{0.1x}}} = \dfrac{10}{1+\dfrac{50\cdot 32}{2^\infty}} = \dfrac{10}{1+0} =$ 10 thousand monthly profit

g) There may not be enough demand to sell a very large number of software packages.

30. $(5,2)$ $t=?$

$x(t) = 4t^3 - 16t^2 + 15t = 5$

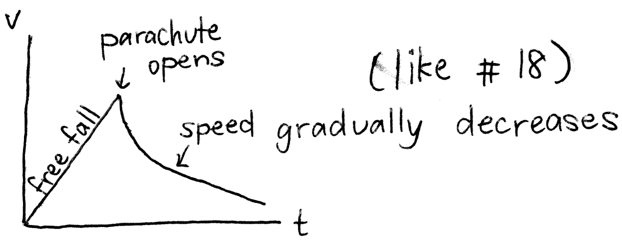
Find intersection on calculator: $t = 2.832$

32. C: position

B: velocity (position always decreasing, so B always − y values)

A: acceleration (B dec then inc, so A − y values then + y values)

33.



(like # 18)

34. $V = \frac{4}{3}\pi r^3$

a) $\frac{dV}{dr} = 4\pi r^2$

$\frac{dV}{dr}(2) = 4\pi \cdot 2^2 = 16\pi$ ft^3 per ft

b) $V(2) = \frac{4}{3}\pi \cdot 2^3 = \frac{32}{3}\pi$

$V(2.2) = \frac{4}{3}\pi \cdot 2.2^3 = 14.197\pi$

$14.197\pi - \frac{32}{3}\pi = 11.092$ ft^3 change in volume

35. $s = v_0 t - 16t^2$

$\frac{ds}{dt} = v_0 - 32t = 0$

$v_0 = 32t$

$t = \frac{v_0}{32}$

Max height = vel 0

$S = V_0 t - 16t^2$

$V_0\left(\frac{V_0}{32}\right) - 16\left(\frac{V_0}{32}\right)^2 = 1900$

$\frac{V_0^2}{32} - \frac{V_0^2}{64} = 1900$

$\frac{V_0^2}{64} = 1900$

$V_0^2 = 121{,}600$

$V_0 = 348.712$ ft/s

$\frac{348.712\ \cancel{ft}}{\cancel{s}} \cdot \frac{1\ mi}{5280\ \cancel{ft}} \cdot \frac{60\ \cancel{s}}{1\ \cancel{min}} \cdot \frac{60\ \cancel{min}}{1\ hr}$

237.758 mi/hr

38. $s = 490t^2$

a) $160 = 490t^2$

$t^2 = \frac{16\cancel{0}}{49\cancel{0}} = \frac{16}{49}$

$t = \sqrt{\frac{16}{49}} = \boxed{\frac{4}{7}\text{ s}}$

Avg. vel. $= \frac{\Delta s}{\Delta t} = \frac{160\text{ cm}}{4/7\text{ s}} = \boxed{280\text{ cm/s}}$

b) $v = 980t$

$v(4/7) = 980(4/7) = \boxed{560\text{ cm/s}}$

$a = \boxed{980\text{ cm/s}^2}$

c) 80 cm = 12 pictures (flashes)

$80 = 490t^2$

$t = \sqrt{\frac{80}{490}} = 0.404\text{ s}$

$\frac{\text{Flashes}}{\text{Second}} = \frac{12\text{ flashes}}{0.404\text{ sec}} \approx \boxed{30\text{ flashes/sec}}$