Section 3.5: 1-33 odd, 38-41 all

1. $y = 1 + x - \cos x$

$\frac{dy}{dx} = 0 + 1 - (-\sin x) = \boxed{1 + \sin x}$

3. $y = \frac{1}{x} + 5\sin x = x^{-1} + 5\sin x$

$\frac{dy}{dx} = -x^{-2} + 5\cos x = \boxed{\frac{-1}{x^2} + 5\cos x}$

5. $y = 4 - x^2 \cdot \sin x$

$\frac{dy}{dx} = -x^2 \cdot \cos x + \sin x(-2x) = \boxed{-x^2\cos x - 2x\sin x}$

7. $y = \frac{4}{\cos x} = 4\sec x$

$\frac{dy}{dx} = \boxed{4\sec x \tan x}$

9. $y = \frac{\cot x}{1 + \cot x}$

$\frac{dy}{dx} = \frac{(1+\cot x)(-\csc^2 x) - (\cot x)(-\csc^2 x)}{(1+\cot x)^2} = \frac{-\csc^2 x\,(1 + \cancel{\cot x} - \cancel{\cot x})}{(1+\cot x)^2} = \boxed{\frac{-\csc^2 x}{(1+\cot x)^2}}$

11. $s = 5\sin t$
$v = 5\cos t$
$a = -5\sin t$

At $t=0$, the weight starts at 0. When $t = \pi/2$, the weight moves to 5. When $t = 3\pi/2$, the weight has moved to -5. The weight oscillates between 5 & -5. Velocity $= 0$ when $t = \pi/2, 3\pi/2, 5\pi/2, 7\pi/2$, etc. Acceleration $= 0$ when $t = 0, \pi, 2\pi, 3\pi$, etc

13. $s = 2 + 3\sin t$

a) $v = 3\cos t$
speed $= |3\cos t|$
$a = -3\sin t$

b) $v(\pi/4) = 3\cos \pi/4 = 3 \cdot \frac{\sqrt{2}}{2} = \boxed{\frac{3\sqrt{2}}{2}}$

speed $= \left|\frac{3\sqrt{2}}{2}\right| = \boxed{\frac{3\sqrt{2}}{2}}$

$a(\pi/4) = -3\sin \pi/4 = -3 \cdot \frac{\sqrt{2}}{2} = \boxed{\frac{-3\sqrt{2}}{2}}$

c) The object starts at position $= 2$ when $t = 0$. When $t = \pi/2$, position $= 5$. When $t = 3\pi/2$, position $= -1$. The object oscillates between 5 & -1.

15. $s = 2\sin t + 3\cos t$

a) $v = 2\cos t - 3\sin t$

speed $= |2\cos t - 3\sin t|$

$a = -2\sin t - 3\cos t$

b) $v(\pi/4) = 2\cos\pi/4 - 3\sin\pi/4 = 2\cdot\frac{\sqrt{2}}{2} - 3\cdot\frac{\sqrt{2}}{2} = \frac{\sqrt{2}}{2}(2-3) = \boxed{-\frac{\sqrt{2}}{2}}$

speed $(\pi/4) = |-\sqrt{2}/2| = \boxed{\sqrt{2}/2}$

$a(\pi/4) = -2\sin\pi/4 - 3\cos\pi/4 = -2\cdot\frac{\sqrt{2}}{2} - 3\cdot\frac{\sqrt{2}}{2} = \frac{\sqrt{2}}{2}(-2-3) = \boxed{-\frac{5\sqrt{2}}{2}}$

c) Graph position on calculator
At $t=0$, the object starts at 3. When $t = 0.588$, position is at a max of 3.606. When $t = 3.730$, position is at a min of -3.606. The position oscillates between 3.606 & -3.606.

17. $s = 2\cos t$

$v = -2\sin t$

$a = -2\cos t$

$j = \boxed{2\sin t}$

19. $s = \sin t - \cos t$

$v = \cos t + \sin t$

$a = -\sin t + \cos t$

$j = \boxed{-\cos t - \sin t}$

21. $y = \sin x + 3$ at $x = \pi$

$y' = \cos x$

$y'(\pi) = \cos\pi = -1$

$y(\pi) = \sin\pi + 3 = 0 + 3 = 3 \rightarrow (\pi, 3)$

Tan: $y - 3 = -1(x - \pi)$

$y - 3 = -x + \pi$

$\boxed{y = -x + \pi + 3}$

Normal: $y - 3 = 1(x - \pi)$

$y - 3 = x - \pi$

$\boxed{y = x - \pi + 3}$

23. $y = x^2 \cdot \sin x$ at $x = 3$

$y(3) = 3^2 \sin 3 = 9\sin 3 \approx 1.270$

$y' = x^2\cos x + 2x\sin x$

$y'(3) = 9\cos 3 + 6\sin 3 \approx -8.063$

$\perp = \frac{-1}{-8.063} = 0.124$

Tan: $y - 1.270 = -8.063(x - 3)$

$y - 1.270 = -8.063x + 24.189$

$\boxed{y = -8.063x + 25.459}$

Normal: $y - 1.270 = 0.124(x - 3)$

$y - 1.270 = 0.124x - 0.372$

$\boxed{y = 0.124x + 0.898}$

25. a) $y = \tan x = \frac{\sin x}{\cos x}$

$$y' = \frac{\cos x \cdot \cos x - \sin x(-\sin x)}{\cos^2 x} = \frac{\cos^2 x + \sin^2 x}{\cos^2 x} = \frac{1}{\cos^2 x} = \boxed{\sec^2 x}$$

b) $y = \sec x = \frac{1}{\cos x}$

$$y' = \frac{\cos x(0) - 1(-\sin x)}{\cos^2 x} = \frac{\cancel{0} + \sin x}{\cos^2 x} = \frac{\sin x}{\cos x} \cdot \frac{1}{\cos x} = \boxed{\tan x \cdot \sec x}$$

27. $y = \sec x$

$y' = \sec x \tan x$

$y'(0) = \sec 0 \cdot \tan 0 = 1 \cdot 0 = \boxed{0}$

$y = \cos x$

$y' = -\sin x$

$y'(0) = -\sin 0 = -0 = \boxed{0}$

29. $y = \sqrt{2}\cos x$ at $(\pi/4, 1)$

$y' = -\sqrt{2}\sin x$

$y'(\pi/4) = -\sqrt{2} \cdot \sin \pi/4 = \frac{-\sqrt{2}}{1} \cdot \frac{\sqrt{2}}{2} = \frac{-\sqrt{4}}{2} = \frac{-2}{2} = -1$

Tan: $y - 1 = -1(x - \pi/4)$

$y - 1 = -x + \pi/4$

$\boxed{y = -x + \pi/4 + 1}$

Normal: $y - 1 = 1(x - \pi/4)$

$y - 1 = x - \pi/4$

$\boxed{y = x - \pi/4 + 1}$

31. $y = 4 + \cot x - 2\csc x$

$y' = -\csc^2 x + 2\csc x \cot x$

a) $y'(\pi/2) = -(\csc \pi/2)^2 + 2\csc \pi/2 \cdot \cot \pi/2 = -1 + 2 \cdot 1 \cdot 0 = -1 + 0 = -1$

$y - 2 = -1(x - \pi/2) \rightarrow y - 2 = -x + \pi/2 \rightarrow \boxed{y = -x + \pi/2 + 2}$

b) $-\csc^2 x + 2\csc x \cot x = 0$

$\csc x(2\cot x - \csc x) = 0$

$\csc x = 0$ when $\sin x =$ undefined $\rightarrow$ never

$2\cot x - \csc x = 0$

$2\cot x = \csc x$

$\frac{2\cos x}{\sin x} = \frac{1}{\sin x} \rightarrow 2\cos x = 1 \rightarrow \cos x = 1/2 \rightarrow x = \pi/3$

$$y(\pi/3) = 4 + \cot \pi/3 - 2\csc \pi/3 = 4 + \frac{1/2}{\sqrt{3}/2} - \frac{2}{\sqrt{3}/2} = 4 + \frac{1}{\sqrt{3}} - \frac{4}{\sqrt{3}} = 4 - \frac{3 \cdot \sqrt{3}}{\sqrt{3} \cdot \sqrt{3}} = 4 - \frac{\cancel{3}\sqrt{3}}{\cancel{3}}$$

$\boxed{y = 4 - \sqrt{3}}$

33. $s = 2 - 2\sin t$

a) $v = -2\cos t$

speed $= |-2\cos t|$

$a = 2\sin t$

$j = 2\cos t$

b) $v(\pi/4) = -\cancel{2}\cdot\frac{\sqrt{2}}{\cancel{2}} = -\sqrt{2}$ m/s

speed $(\pi/4) = |-\sqrt{2}| = \sqrt{2}$ m/s

$a(\pi/4) = \cancel{2}\cdot\frac{\sqrt{2}}{\cancel{2}} = \sqrt{2}$ m/s^2

$j(\pi/4) = \cancel{2}\cdot\frac{\sqrt{2}}{\cancel{2}} = \sqrt{2}$ m/s^3

c) At $t=0$, the position is 2. At $t=\pi/2$, the object has moved down to 0. At $t=3\pi/2$, the object has moved up to 4. The motion oscillates between 0 & 4.

38. 999^{th} derivative of $\cos x$

$999 = 4\times 249 + 3$

$\cos x$
$-\sin x$
$-\cos x$
$\boxed{\sin x}$

$\cos x$
$-\sin x$
$-\cos x$
$\sin x$

39. 725^{th} derivative of $\sin x$

$725 = 4 \times 181 + 1$

$\sin x$
$\boxed{\cos x}$

$\sin x$
$\cos x$
$-\sin x$
$-\cos x$

40. $y = x$

41. a) $\sin x \approx x$

$\sin 0.12 \approx 0.12$

b) $\sin 0.12 = 0.1197122073\ldots$

$0.12 - 0.1197122073 =$ within 0.0003 of the true value