Trig Function Derivatives (Section 3.5)

** $y = \sin x \rightarrow y' = \cos x$
$y = \cos x \rightarrow y' = -\sin x$
$y = \tan x \rightarrow y' = \sec^2 x$

} MUST be Memorized!

$y = \csc x \rightarrow y' = -\csc x \cdot \cot x$
$y = \sec x \rightarrow y' = \sec x \cdot \tan x$
$y = \cot x \rightarrow y' = -\csc^2 x$

Not used as much, but useful to know!

ex: $y = 2\sin x - \tan x$

$y' = 2(\cos x) - (\sec^2 x)$

$\boxed{y' = 2\cos x - \sec^2 x}$

ex: $y = 3x + x\tan x$

$y' = 3 + x(\sec^2 x) + \tan x(1)$

$\boxed{y' = 3 + x\sec^2 x + \tan x}$

ex: Prove $\frac{d}{dx}\cot x = -\csc^2 x$

$\frac{d}{dx}\left(\frac{\cos x}{\sin x}\right) =$

$= \frac{\sin x(-\sin x) - \cos x(\cos x)}{(\sin x)^2}$

$= \frac{-\sin^2 x - \cos^2 x}{(\sin x)^2}$

$= \frac{-1(\sin^2 x + \cos^2 x)}{\sin^2 x}$ ($\sin^2 x + \cos^2 x = 1$)

$= \frac{-1}{\sin^2 x} = -\csc^2 x$ ✓

ex: $y = \frac{\cos x}{1 + \sin x}$

$y' = \frac{(1+\sin x)(-\sin x) - (\cos x)(\cos x)}{(1+\sin x)^2}$

$= \frac{-\sin x - \sin^2 x - \cos^2 x}{(1+\sin x)^2}$

$= \frac{-\sin x - 1(\sin^2 x + \cos^2 x)}{(1+\sin x)^2}$ ($\sin^2 x + \cos^2 x = 1$)

$= \frac{-\sin x - 1}{(1+\sin x)^2} = \frac{-1(\sin x + 1)}{(1+\sin x)^2}$

$\boxed{= \frac{-1}{1+\sin x}}$

** Remember:

$\sin^2 x + \cos^2 x = 1$

$\sin 2x = 2\sin x \cos x$

$\cos 2x = \cos^2 x - \sin^2 x$

all are on pg. 573

#22 Find eq. for the lines that are tangent & normal to $y = \sec x$ @ $x = \pi/4$. → $(\pi/4, \sqrt{2})$

$y' = \sec x \cdot \tan x$

$y'(\pi/4) = \sec(\pi/4) \cdot \tan(\pi/4)$

$= (\sqrt{2}) \cdot (1)$

$y' = \sqrt{2}$

$m = \sqrt{2}$

$\perp m = \frac{-1}{\sqrt{2}}$

Tangent

$y - \sqrt{2} = \sqrt{2}(x - \pi/4)$

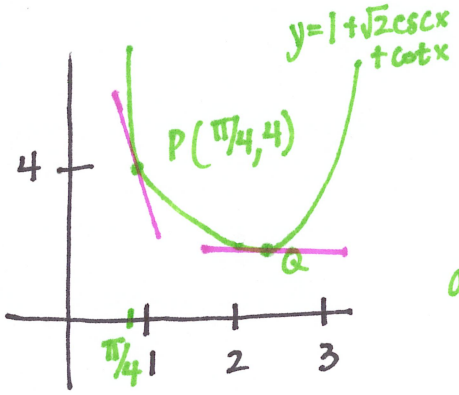
$\approx y = 1.414x + .303$

Normal

$y - \sqrt{2} = \frac{-1}{\sqrt{2}}(x - \pi/4)$

$\approx y = -.707x + 1.970$

#32



a) Find the tangent line @ point P.

b) Find the horizontal tangent @ point Q.

a) $y = 1 + \sqrt{2}\csc x + \cot x$

$y' = \sqrt{2}(-\csc x \cot x) - \csc^2 x$ @ $(\pi/4, 4)$

$y'(\pi/4) = -\sqrt{2}(\sqrt{2})(1) - (\sqrt{2})^2$

$= -2 - 2$

$= -4$

$m = -4$ @ $(\pi/4, 4)$ → $y - 4 = -4(x - \pi/4)$

$\boxed{y = -4x + 4 + \pi}$

b) $y' = -\sqrt{2}\csc x \cot x - \csc^2 x$

Horz Tangent → $y' = 0$

$-\csc x(\sqrt{2}\cot x + \csc x) = 0$

$-\csc x = 0$

$\sqrt{2} \cdot \frac{\cos x}{\sin x} + \frac{1}{\sin x} = 0$

$\frac{\sqrt{2}\cos x + 1}{\sin x} = 0$

$\sqrt{2}\cos x + 1 = 0$

$\cos x = -1/\sqrt{2}$

$x = 3\pi/4$

$y(3\pi/4) = 1 + \sqrt{2}\csc(3\pi/4) + \cot(3\pi/4)$

$= 1 + \sqrt{2}(\sqrt{2}) + (-1)$

$= 1 + 2 - 1$ $\boxed{= 2}$