Section 4.1 Key - Day 1: 1-49 e.o.o., 51-55 odd

1. $y = \sin(3x+1)$, $u = 3x+1$

$y' = \cos u \, du = \cos(3x+1)\cdot 3 = 3\cos(3x+1)$

5. $y = \left(\frac{\sin x}{1+\cos x}\right)^2$, $u = \frac{\sin x}{1+\cos x}$

$du = \frac{(1+\cos x)\cos x - \sin x(-\sin x)}{(1+\cos x)^2} = \frac{\cos x + \cos^2 x + \sin^2 x}{(1+\cos x)^2} = \frac{1+\cos x}{(1+\cos x)^2} = \frac{1}{1+\cos x}$

$y' = 2u\,du = 2\cdot\frac{\sin x}{1+\cos x}\cdot\frac{1}{1+\cos x} = \frac{2\sin x}{(1+\cos x)^2}$

9. $s = \cos(\pi/2 - 3t)$

$s' = -\sin(\pi/2 - 3t)(-3) = 3\sin(\pi/2 - 3t)$

13. $y = (x+\sqrt{x})^{-2}$

$y' = -2(x+\sqrt{x})^{-3}(1+\tfrac{1}{2}x^{-1/2}) = \frac{-2-x^{-1/2}}{(x+\sqrt{x})^3} = \frac{-2-\frac{1}{\sqrt{x}}}{(x+\sqrt{x})^3}$

17. $y = (\sin x)^3 \cdot \tan 4x$

$y' = (\sin x)^3 \cdot \sec^2 4x \cdot 4 + \tan 4x \cdot 3\sin^2 x \cos x$

21. $y = (\sin(3x-2))^2$

$y' = 2\sin(3x-2)\cdot\cos(3x-2)\cdot 3 = 6\sin(3x-2)\cos(3x-2)$

25. $r = \tan(2-\theta)$

$\frac{dr}{d\theta} = \sec^2(2-\theta)(-1) = -\sec^2(2-\theta)$

29. $y = \tan x$

$y' = \sec^2 x = (\sec x)^2$

$y'' = 2\sec x \cdot \sec x \tan x = 2\sec^2 x \tan x$

33. $f(g(x)) = \sqrt{x}^5 + 1 = x^{5/2} + 1$ at $x = 1$

$(f(g(x)))' = \frac{5}{2} x^{3/2}$

$(f(g(1)))' = \frac{5}{2} \cdot 1^{3/2} = \frac{5}{2}$

37. $f(u) = \frac{2u}{u^2+1}$, $u = g(x) = 10x^2 + x + 1$ at $x = 0 \rightarrow u(0) = 0 + 0 + 1 = 1$

$du = 20x + 1$

When $x = 0$, $u = 1$

$(f \circ g)' = f'(u)\, du$

$f'(u) = \frac{(u^2+1)2 - 2u(2u)}{(u^2+1)^2} = \frac{2u^2 + 2 - 4u^2}{(u^2+1)^2} = \frac{2 - 2u^2}{(u^2+1)^2}$

$(f \circ g)' = f'(u)\, du = \frac{2 - 2u^2}{(u^2+1)^2} \cdot (20x + 1)$

$(f \circ g)'(0) = \frac{2 - 2(1)^2}{(1^2+1)^2} \cdot (20(0) + 1) = \frac{2-2}{2^2}(0+1) = \frac{0(1)}{4} = 0$

41. $x = 2\cos t$, $y = 2\sin t$, $t = \pi/4$

$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{2\cos t}{-2\sin t} = -\cot t$

$\frac{dy}{dx}(\pi/4) = -\cot(\pi/4) = -1$

$x(\pi/4) = 2\cos \pi/4 = 2 \cdot \sqrt{2}/2 = \sqrt{2}$

$y(\pi/4) = 2\sin \pi/4 = 2 \cdot \sqrt{2}/2 = \sqrt{2}$

$> (\sqrt{2}, \sqrt{2})$

$y - \sqrt{2} = -1(x - \sqrt{2})$

$y - \sqrt{2} = -x + \sqrt{2}$

$y = -x + 2\sqrt{2}$

45. $x = t$, $y = \sqrt{t} = t^{1/2}$, $t = 1/4$

$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{\frac{1}{2} t^{-1/2}}{1} = \frac{1}{2\sqrt{t}}$

$\frac{dy}{dx}\left(\frac{1}{4}\right) = \frac{1}{2\sqrt{1/4}} = \frac{1}{2 \cdot 1/2} = \frac{1}{1} = 1$

$x(1/4) = 1/4$

$y(1/4) = \sqrt{1/4} = 1/2$

$> (1/4, 1/2)$

$y - \frac{1}{2} = 1(x - 1/4)$

$y - \frac{1}{2} = x - \frac{1}{4}$

$y = x + \frac{1}{4}$

49. $x = t^2 + t, \quad y = \sin t$

a) $\dfrac{dy}{dx} = \dfrac{dy/dt}{dx/dt} = \dfrac{\cos t}{2t+1}$

b) $\dfrac{d}{dt}\left(\dfrac{dy}{dx}\right) =$ derivative of dy/dx with respect to t

$$\frac{(2t+1)(-\sin t) - \cos t \cdot 2}{(2t+1)^2} = \frac{-2t\sin t - \sin t - 2\cos t}{(2t+1)^2}$$

c) $u = \dfrac{dy}{dx}$

$$\frac{du}{dt} = \frac{du}{dx} \cdot \frac{dx}{dt}$$

$$\frac{du}{dx} = \frac{du}{dt} \div \frac{dx}{dt}$$

$$\frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dt}\left(\frac{dy}{dx}\right) \div \frac{dx}{dt} = \frac{-2t\sin t - \sin t - 2\cos t}{(2t+1)^2} \cdot \frac{1}{2t+1} = \frac{-2t\sin t - \sin t - 2\cos t}{(2t+1)^3}$$

d) $\dfrac{d^2y}{dx^2} = \dfrac{d}{dx}\left(\dfrac{dy}{dx}\right) =$ Part C

51. $s = \cos\theta, \quad \theta = 3\pi/2, \quad d\theta/dt = 5$

$$\frac{ds}{d\theta} = -\sin\theta \cdot \frac{d\theta}{dt} = -\sin\frac{3\pi}{2} \cdot 5 = -(-1) \cdot 5 = +5$$

53. $y = \sin\left(\frac{1}{2}x\right)$

$y' = \frac{1}{2}\cos\left(\frac{1}{2}x\right)$

Max value of $\cos\left(\frac{1}{2}x\right)$ is 1, so $\frac{1}{2} \cdot 1 = \frac{1}{2}$ maximum slope

55. $y = 2\tan\left(\frac{\pi}{4}x\right)$ at $(1,2)$

$y' = 2\sec^2\left(\frac{\pi}{4}x\right) \cdot \frac{\pi}{4} = \frac{\pi}{2}\left(\sec\frac{\pi}{4}x\right)^2$

$y'(1) = \frac{\pi}{2}\left(\sec\frac{\pi}{4}\right)^2 = \frac{\pi}{2} \cdot \sqrt{2}^2 = \frac{\pi}{2} \cdot 2 = \pi$

Tangent: $y - 2 = \pi(x-1)$

$y - 2 = \pi x - \pi$

$y = \pi x - \pi + 2$

Normal: $y - 2 = \frac{-1}{\pi}(x-1)$

$y - 2 = -\frac{1}{\pi}x + \frac{1}{\pi}$

$y = \frac{-1}{\pi}x + \frac{1}{\pi} + 2$