Section 4.1 Key – Day 2: 3–47 e.o.o, 58–66 even

3. $y=\cos(\sqrt{3}x)$, $u=\sqrt{3}x$

$y'=f'(u)du \qquad du=\sqrt{3}$

$y'=-\sin(\sqrt{3}x)\cdot\sqrt{3} = -\sqrt{3}\sin(\sqrt{3}x)$

7. $y=\cos(\sin x)$, $u=\sin x$

$du=\cos x$

$y'=f'(u)\,du$

$y'=-\sin(\sin x)\cdot\cos x$

11. $s=\frac{4}{3\pi}\sin 3t+\frac{4}{5\pi}\cos 5t$

$v=\frac{4}{3\pi}\cos 3t\cdot 3+\frac{-4}{5\pi}\sin 5t\cdot 5 = \frac{4}{\pi}\cos 3t-\frac{4}{\pi}\sin 5t$

15. $y=\sin^{-5}x-\cos^3 x$

$y'=-5\sin^{-6}x\cdot\cos x-3\cos^2 x(-\sin x) = \frac{-5\cos x}{\sin^6 x}+3\cos^2 x\sin x$

19. $y=3(2x+1)^{-1/2}$

$y'=-\frac{3}{\cancel{2}}(2x+1)^{-3/2}\cdot\cancel{2} = \frac{-3}{(2x+1)^{3/2}}$

23. $y=(1+\cos^2 7x)^3$

$y'=3(1+\cos^2 7x)^2\cdot 2\cos 7x\cdot -\sin 7x\cdot 7 = -42\sin 7x\cos 7x(1+\cos^2 7x)^2$

27. $r=(\theta\sin\theta)^{1/2}$

$\frac{dr}{d\theta}=\frac{1}{2}(\theta\sin\theta)^{-1/2}(\theta\cos\theta+\sin\theta\cdot 1) = \frac{\theta\cos\theta+\sin\theta}{2\sqrt{\theta\sin\theta}}$

31. $y=\cot(3x-1)$

$y'=-\csc^2(3x-1)\cdot 3 = -3(\csc(3x-1))^2$

$y''=-6(\csc(3x-1))\cdot -\csc(3x-1)\cot(3x-1)\cdot 3 = 18\csc^2(3x-1)\cot(3x-1)$

35. $f(u) = \cot\left(\frac{\pi}{10}u\right)$, $u = 5\sqrt{x}$, $du = \frac{5}{2}x^{-1/2}$, $x = 1$ $\quad u(1) = 5\sqrt{1} = 5$

$(f\circ g)' = f'(u)\,du$

$f'(u) = -\csc^2\left(\frac{\pi}{10}u\right)\cdot\frac{\pi}{10} = -\frac{\pi}{10}\csc^2\left(\frac{\pi}{10}u\right)$

$(f\circ g)' = f'(u)\,du = -\frac{\pi}{10}\csc^2\left(\frac{\pi}{10}\underset{5}{u}\right)\cdot\frac{5}{2\sqrt{x}_{\,1}} = \frac{-\pi}{10}\csc^2\left(\frac{\pi}{2}\right)\cdot\frac{5}{2} = -\frac{\pi}{10}\cdot\cancel{1}^{\,2}\cdot\frac{5}{2} = \frac{-5\pi}{20} = \frac{-\pi}{4}$

39. $y = \cos(6x+2)$

a) $y = \cos u$, $u = 6x+2 \to du = 6$

$y' = f'(u)\,du$

$y' = -\sin(6x+2)\cdot 6 = -6\sin(6x+2)$

b) $y = \cos 2u$, $u = 3x+1 \to du = 3$

$y' = f'(u)\,du$

$y' = -\sin 2u\cdot 2\cdot 3 = -6\sin 2(3x+1) = -6\sin(6x+2)$

43. $x = \sec^2 t - 1$, $y = \tan t$, $t = -\pi/4$

$x(-\pi/4) = (\sec(-\pi/4))^2 - 1 = \sqrt{2}^{\,2} - 1 = 2 - 1 = 1$

$y(-\pi/4) = \tan(-\pi/4) = -1$ $\quad > (1,-1)$

$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{\cancel{\sec^2 t}}{2\cancel{\sec t}\cdot\cancel{\sec t}\cdot\tan t} = \frac{\cot t}{2} = \frac{\cot(-\pi/4)}{2} = \frac{-1}{2}$

$y+1 = -\frac{1}{2}(x-1)$

$y+1 = -\frac{1}{2}x + \frac{1}{2}$

$y = \frac{-1}{2}x - \frac{1}{2}$

47. $x = t - \sin t$, $y = 1 - \cos t$, $t = \pi/3$

$x(\pi/3) = \frac{\pi}{3} - \sin\frac{\pi}{3} = \frac{\pi}{3} - \frac{\sqrt{3}}{2}$

$y(\pi/3) = 1 - \cos\frac{\pi}{3} = 1 - \frac{1}{2} = \frac{1}{2}$ $\quad > \left(\frac{\pi}{3} - \frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

$\frac{dy}{dx} = \frac{dy/dx}{dx/dt} = \frac{\sin t}{1-\cos t} = \frac{\sin \pi/3}{1-\cos\pi/3} = \frac{\sqrt{3}/2}{1-1/2} = \frac{\sqrt{3}/2}{1/2} = \frac{\sqrt{3}}{\cancel{2}}\cdot\frac{\cancel{2}}{1} = \sqrt{3}$

$y - \frac{1}{2} = \sqrt{3}\left(x - \frac{\pi}{3} + \frac{\sqrt{3}}{2}\right)$

$y - \frac{1}{2} = \sqrt{3}x - \frac{\sqrt{3}}{3}\pi + \frac{3}{2}$

$y = \sqrt{3}x - \frac{\sqrt{3}}{3}\pi + 2$

58. a) $5f(x) - g(x)$, $x = 1$

$5f'(x) - g'(x) = 5f'(1) - g'(1) = 5(-1/3) - -8/3 = -5/3 + 8/3 = 3/3 = 1$

b) $f(x)\cdot g^3(x)$, $x = 0$

$f(x)\cdot 3g^2(x)\,g'(x) + g^3(x)\cdot f'(x) = 1\cdot\cancel{3}\cdot 1^2\cdot\cancel{1/3} + 1^3\cdot 5 = 1+5 = 6$

58. c) $\frac{f(x)}{g(x)+1}$, $x=1$

$$\frac{(g(x)+1)f'(x)-f(x)\cdot g'(x)}{(g(x)+1)^2} = \frac{(-4+1)(-1/3)-3\cdot -8/3}{(-4+1)^2} = \frac{1+8}{9} = 1$$

d) $f(g(x))$, $x=0$

$$f'(g(x))\cdot g'(x) = f'(g(0))\cdot g'(0) = f'(1)\cdot g'(0) = -1/3\cdot 1/3 = -1/9$$

e) $g(f(x))$, $x=0$

$$g'(f(x))\cdot f'(x) = g'(f(0))\cdot f'(0) = g'(1)\cdot f'(0) = -8/3\cdot 5 = -40/3$$

f) $(g(x)+f(x))^{-2}$, $x=1$

$$-2(g(x)+f(x))^{-3}(g'(x)+f'(x)) = \frac{-2(-8/3+-1/3)}{(-4+3)^3} = \frac{-2(-3)}{(-1)^3} = \frac{6}{-1} = -6$$

g) $f(x+g(x))$, $x=0$

$$f'(x+g(x))(1+g'(x)) = f'(g(0))\cdot(1+g'(0)) = f'(1)(1+1/3) = -1/3\cdot 4/3 = -4/9$$

60. $\frac{dy}{dx} = \frac{dy}{du}\cdot\frac{du}{dx}$ looks like multiplying fractions, but each part is the notation for a derivative, not a fraction. dy/dx is the deriv. of y w/ respect to x. dy/du is the deriv. of y w/ respect to u. du/dx is the deriv. of u w/ respect to x.

62. $y = 37\sin\left(\frac{2\pi}{365}x - \frac{202\pi}{365}\right) + 25$

a) Temp increasing fastest = max + slope

$$y' = \underbrace{37\cos\left(\frac{2\pi}{365}x - \frac{202\pi}{365}\right)}_{\text{At most } +1}\cdot\frac{2\pi}{365}$$

Max + cos value at 0.

$$\frac{2\pi}{365}x - \frac{202\pi}{365} = 0$$

$$\frac{2\cancel{\pi}x}{\cancel{365}} = \frac{202\cancel{\pi}}{\cancel{365}}$$

$$2x = 202$$

$x = 101 \rightarrow$ $\boxed{101^{st}\text{ day, April 11}}$

b) Max $y' = 37\cdot 1\cdot\frac{2\pi}{365} = \boxed{0.637\ °F/day}$

64. $v = k\sqrt{s}$ m/s

$v = ks^{1/2}$

$a = \frac{1}{2}ks^{-1/2} \cdot \underset{\uparrow \atop v}{s'} = \frac{k}{2\cancel{\sqrt{s}}} \cdot \underset{\uparrow \atop v}{k\cancel{\sqrt{s}}} = \boxed{\frac{k^2}{2}}$ = constant ✓

66. $\frac{dx}{dt} = f(x)$ = velocity, so x = position

$a = f'(x) \cdot \underset{\uparrow \atop v}{\frac{dx}{dt}} = \boxed{f'(x) \cdot \underset{\uparrow \atop v}{f(x)}}$