Section 4.3 Key: 1, 3, 6, 7, 8, 27, 31, 41, 45

1. $y = \cos^{-1}(x^2)$

$$\frac{dy}{dx} = \frac{-1}{\sqrt{1-(x^2)^2}} \cdot 2x = \boxed{\frac{-2x}{\sqrt{1-x^4}}}$$

3. $y = \sin^{-1}\sqrt{2}\,t$

$$\frac{dy}{dt} = \frac{1}{\sqrt{1-(\sqrt{2}t)^2}} \cdot \sqrt{2} = \boxed{\frac{\sqrt{2}}{\sqrt{1-2t^2}}}$$

6. $y = s\sqrt{1-s^2} + \cos^{-1}s = s(1-s^2)^{1/2} + \cos^{-1}s$

$$\frac{dy}{ds} = s \cdot \tfrac{1}{2}(1-s^2)^{-1/2}(-2s) + (1-s^2)^{1/2} \cdot 1 + \frac{-1}{\sqrt{1-s^2}}$$

$$\frac{dy}{ds} = \frac{-s^2}{\sqrt{1-s^2}} + \sqrt{1-s^2} - \frac{1}{\sqrt{1-s^2}} = \frac{\sqrt{1-s^2}}{1} - \frac{s^2+1}{\sqrt{1-s^2}}$$

$$\frac{dy}{ds} = \frac{\sqrt{1-s^2}\cdot\sqrt{1-s^2}}{\sqrt{1-s^2}} - \frac{s^2+1}{\sqrt{1-s^2}} = \frac{1-s^2-s^2-1}{\sqrt{1-s^2}} = \boxed{\frac{-2s^2}{\sqrt{1-s^2}}}$$

7. $y = x\sin^{-1}x + (1-x^2)^{1/2}$

$$\frac{dy}{dx} = x \cdot \frac{1}{\sqrt{1-x^2}} + \sin^{-1}x \cdot 1 + \tfrac{1}{2}(1-x^2)^{-1/2} \cdot -2x$$

$$\frac{dy}{dx} = \frac{x}{\sqrt{1-x^2}} + \sin^{-1}x - \frac{x}{\sqrt{1-x^2}} = \boxed{\sin^{-1}x}$$

8. $y = \dfrac{1}{\sin^{-1}2x} = (\sin^{-1}2x)^{-1}$

$$\frac{dy}{dx} = -1(\sin^{-1}2x)^{-2} \cdot \frac{1}{\sqrt{1-(2x)^2}} \cdot 2 = \boxed{\frac{-2}{\sqrt{1-4x^2}\,(\sin^{-1}2x)^2}}$$

27. a) $y = \tan x$ at $(\pi/4, 1)$

$\frac{dy}{dx} = \sec^2 x$ at $\pi/4 = (\sec \pi/4)^2 = (\sqrt{2})^2 = 2$

$y - 1 = 2(x - \pi/4)$

$y - 1 = 2x - \pi/2$

$\boxed{y = 2x - \pi/2 + 1}$

b) $y = \tan^{-1} x$ at $(1, \pi/4)$

$\frac{dy}{dx} = \frac{1}{x^2+1}$ at $\pi/4 = \frac{1}{1^2+1} = \frac{1}{2}$

$y - \pi/4 = 1/2\,(x - 1)$

$y - \pi/4 = 1/2\,x - 1/2$

$\boxed{y = \frac{1}{2}x - \frac{1}{2} + \frac{\pi}{4}}$

31. $x = \tan^{-1} t$

a) $v = \frac{1}{t^2+1}$ is always positive → always moving right

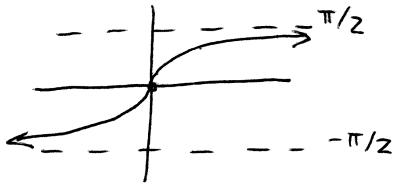
b) $v = (t^2+1)^{-1}$

$a = -(t^2+1)^{-2} \cdot 2t = \frac{-2t}{(t^2+1)^2} = \frac{-2(+)}{(+)^2} = -$

Acceleration negative → always decelerating

c) tant : D: $(-\pi/2, \pi/2)$, R: $(-\infty, \infty)$

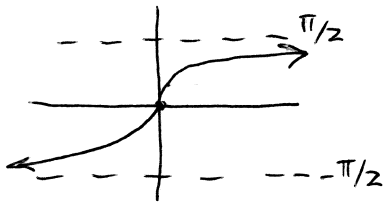
$\tan^{-1} t$: D: $(-\infty, \infty)$, R: $(-\pi/2, \pi/2)$



$\lim_{t \to \infty} \tan^{-1} t = \tan^{-1} \infty = \boxed{\pi/2}$

41. $y = \tan^{-1} x$

$\frac{dy}{dx} = \frac{1}{x^2+1}$



a) $y = \pi/2$

b) $y = -\pi/2$

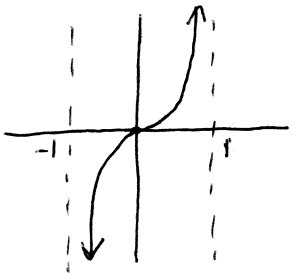
c) $\frac{1}{x^2+1} \neq 0$ → None

45. $y = \sin^{-1} x$

$\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}}$

sinx : D: $(-\infty, \infty)$, R: $[-1, 1]$

$\sin^{-1} x$: D: $[-1, 1]$, R: $(-\infty, \infty)$



a) ∞ → None

b) $-\infty$ → None

c) $\frac{1}{\sqrt{1-x^2}} \neq 0$ → None