Section 4.4 : 1-41 e.o.o., 51-55 odd

1. $y = 2e^x$

$\frac{dy}{dx} = \boxed{2e^x}$

5. $y = e^{2/3x}$

$\frac{dy}{dx} = \boxed{\frac{2}{3}e^{2/3x}}$

9. $y = e^{\sqrt{x}}$

$\frac{dy}{dx} = e^{\sqrt{x}} \cdot \frac{1}{2}x^{-1/2} = \boxed{\frac{e^{\sqrt{x}}}{2\sqrt{x}}}$

13. $y = 3^{\csc x}$

$\frac{dy}{dx} = 3^{\csc x} \cdot \ln 3 \cdot -\csc x \cot x = \boxed{-3^{\csc x} \cdot \ln 3 \cdot \csc x \cot x}$

17. $y = \ln(1/x) = \ln(x^{-1})$

$\frac{dy}{dx} = \frac{1}{1/x} \cdot -1x^{-2} = \frac{-1}{x^{\cancel{2}}} \cdot \frac{x}{1} = \boxed{\frac{-1}{x}}$

21. $y = \log_4 x^2$

$\frac{dy}{dx} = \frac{1}{x^{\cancel{2}} \cdot \ln 4} \cdot 2\cancel{x} = \boxed{\frac{2}{\ln 4\, x}}$

25. $y = \ln 2 \cdot \log_2 x$

$\frac{dy}{dx} = \frac{\cancel{\ln 2}}{x \cdot \cancel{\ln 2}} = \boxed{\frac{1}{x}}$

29. $y = 3^x + 1$ parallel to $y = 5x - 1 \rightarrow m = 5$

$\frac{dy}{dx} = 3^x \cdot \ln 3 = 5$

$3^x = \frac{5}{\ln 3}$

$x \cdot \ln 3 = \ln\left(\frac{5}{\ln 3}\right)$

$x = \frac{\ln(5/\ln 3)}{\ln 3} \approx 1.379$

$y = 3^{1.379} + 1 \approx 5.551$

$\boxed{(1.379, 5.551)}$

33. $y = x^{\pi}$

$$\frac{dy}{dx} = \boxed{\pi x^{\pi - 1}}$$

37. $f(x) = \ln(x+2) \rightarrow x+2 > 0 \rightarrow \boxed{x > -2}$

$$f'(x) = \boxed{\frac{1}{x+2}}$$

41. $f(x) = \log_2(3x+1) \rightarrow 3x+1 > 0 \rightarrow \boxed{x > -1/3}$

$$f'(x) = \frac{1}{(3x+1)\ln 2} \cdot 3 = \boxed{\frac{3}{(3x+1)\ln 2}}$$

51. $P(t) = \dfrac{300}{1+2^{4-t}} = 300(1+2^{4-t})^{-1}$

a) $P(0) = \dfrac{300}{1+2^{4-0}} = \dfrac{300}{1+2^4} = \dfrac{300}{17} \approx 17.647 \rightarrow \boxed{18 \text{ students}}$

b) $P'(t) = \cancel{-}300(1+2^{4-t})^{-2} \cdot 2^{4-t} \cdot \ln 2 \cdot \cancel{-1}$

$P'(t) = \dfrac{300 \cdot \ln 2 \cdot 2^{4-t}}{(1+2^{4-t})^2}$ at $t=4$ $= \dfrac{300 \cdot \ln 2 \cdot \cancel{2^0}}{(1+2^0)^2} = \dfrac{300 \cdot \ln 2}{4} \approx 51.986 \rightarrow \boxed{52 \text{ students/day}}$

c) Graph $P'(t)$ & find max : Time $= \boxed{4 \text{ days}}$, Rate $= \boxed{52 \text{ students/day}}$

53. $A = 20 \cdot (1/2)^{\frac{1}{140}t}$

$A'(t) = 20 \cdot (1/2)^{\frac{1}{140}t} \cdot \ln(1/2) \cdot \dfrac{1}{140} = \dfrac{\ln(1/2) \cdot (1/2)^{1/140t}}{7}$

$A'(2) = \dfrac{\ln(1/2) \cdot (1/2)^{1/70}}{7} = -0.0980 \rightarrow$ decaying at a rate of $\boxed{0.0980 \text{ grams/day}}$

55. $f(x) = 2^x$

a) $f'(x) = 2^x \cdot \ln 2$

$f'(0) = 2^0 \cdot \ln 2 = 1 \cdot \ln 2 = \boxed{\ln 2}$

b) $f'(0) = \lim\limits_{h \to 0} \dfrac{f(0+h) - f(0)}{h} = \lim\limits_{h \to 0} \dfrac{2^{0+h} - 2^0}{h} = \boxed{\lim\limits_{h \to 0} \dfrac{2^h - 1}{h}}$

c) $\lim\limits_{h \to 0} \dfrac{2^h - 1}{h}$ = Derivative of 2^x at $x = 0$ = $\boxed{\ln 2}$ (from part A)

d) $\lim\limits_{h \to 0} \dfrac{7^h - 1}{h}$ = Derivative of 7^x at $x = 0$ $\rightarrow$ $f'(x) = 7^x \cdot \ln 7$

$f'(0) = 7^0 \cdot \ln 7 = 1 \cdot \ln 7 = \boxed{\ln 7}$