Section 5.1: 1-33 e.o.o., 35-43 odd, 53, 55

1. Max: 2 at $x=0$
   Min: 0 at $x=\pm 2$

5. Abs. max at $x=b$
   Abs. min at $x=c_2$
   $h(x)$ is continuous on $[a,b]$, so both a max & min exist.

9. Abs. max at $x=c$
   Abs. min at $x=a$
   $g(x)$ is not continuous on $[a,b]$, so extreme value theorem doesn't apply.

13. $h(x)=\ln(x+1),\ 0\le x\le 3$

$h(0)=\ln(0+1)=\ln 1=0 \to (0,0)$

$h(3)=\ln(3+1)=\ln 4 \to (3,\ln 4)$

$h'(x)=\dfrac{1}{x+1}\neq 0$, but undefined at $x=-1$ ← but not in domain

$(0,0) \to$ Min of 0 at $x=0$ (End)
$(3,\ln 4) \to$ Max of $\ln 4$ at $x=3$ (End)

17. $f(x)=x^{2/5},\ -3\le x<1$

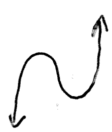
$f(-3)=(-3)^{2/5}=\sqrt[5]{9} \to (-3,\sqrt[5]{9})$

$f'(x)=\frac{2}{5}x^{-3/5}=\dfrac{2}{5x^{3/5}}\neq 0$, but undefined at $x=0$

$f(0)=0^{2/5}=0 \to (0,0)$

$(-3,\sqrt[5]{9}) \to$ Max of $\sqrt[5]{9}$ at $x=-3$ (End)
$(0,0) \to$ Min of 0 at $x=0$
(Critical Pt., not Stationary)

21. $y=x^3+x^2-8x+5$



No endpoints

$y'=3x^2+2x-8$ is not undefined

$3x^2+2x-8=0$

$3x^2+6x-4x-8=0$

$3x(x+2)-4(x+2)=0$

$(x+2)(3x-4)=0$

$y'=0$ at $x=-2,\ x=4/3$

$y(-2)=(-2)^3+(-2)^2-8(-2)+5$

$y(-2)=-8+4+16+5=17 \to (-2,17)$

$y(4/3)=(4/3)^3+(4/3)^2-8(4/3)+5$

$y(4/3)=\frac{64}{27}+\frac{16}{9}-\frac{32}{3}+\frac{5}{1}$

$y(4/3)=\frac{64}{27}+\frac{48}{27}-\frac{288}{27}+\frac{135}{27}=\frac{-41}{27} \to \left(\frac{4}{3},\frac{-41}{27}\right)$

Local Max of 17 at $x=-2$ (CP, S)
Local Min of $-41/27$ at $x=4/3$ (CP, S)

25. $y = \frac{1}{\sqrt{1-x^2}}$ $\quad 1-x^2>0$

$x^2<1$

$(-1,1)$ domain, no endpoints

$y = (1-x^2)^{-1/2}$

$y' = -\cancel{\tfrac{1}{2}}(1-x^2)^{-3/2} \cdot -\cancel{2}x = \frac{x}{(1-x^2)^{3/2}} = 0$ at $x=0$

y' is undefined at $x=\pm 1$, but the values are not in the domain of $f(x)$.

$y(0) = \frac{1}{\sqrt{1-0^2}} = \frac{1}{\sqrt{1}} = 1 \to (0,1)$

Check another point in domain $(-1,1)$ to see if $(0,1)$ is a min or max.

$y(1/2) = \frac{1}{\sqrt{1-(1/2)^2}} = \frac{1}{\sqrt{1-1/4}} = \frac{1}{\sqrt{3/4}} \approx 1.155 \to (1/2, 1.155)$

↑ greater than $y=1$

$\boxed{\text{Min of 1 at } x=0 \text{ (CP, S)}}$

29. $y = \frac{x}{x^2+1}$

No endpoints, unlimited domain

$y' = \frac{(x^2+1)(1)-(x)(2x)}{(x^2+1)^2} = \frac{x^2+1-2x^2}{(x^2+1)^2} = \frac{-x^2+1}{(x^2+1)^2}$

$y'=0$ when $-x^2+1=0 \to x^2=1 \to x=\pm 1$

y' is never undefined bc $(x^2+1)^2 \neq 0$.

$y(1) = \frac{1}{1^2+1} = \frac{1}{2} \to (1, 1/2) \to$ $\boxed{\text{Max of } 1/2 \text{ at } x=1 \text{ (CP, S)}}$

$y(-1) = \frac{-1}{(-1)^2+1} = \frac{-1}{2} \to (-1,-1/2) \to$ $\boxed{\text{Min of } -1/2 \text{ at } x=-1 \text{ (CP, S)}}$

33. $h(x) = |x+2|-|x-3|$, $-\infty < x < \infty \to$ no endpoints

Graph in calculator

$\boxed{\begin{array}{l}\text{Max of 5 on } [3,\infty) \\ \text{Min of } -5 \text{ on } (-\infty,-2]\end{array}}$

35. $y = x^{2/3}(x+2) = x^{5/3}+2x^{2/3}$, no endpoints

$y' = \frac{5}{3}x^{2/3} + \frac{4}{3}x^{-1/3} = \frac{5}{3}x^{2/3} + \frac{4}{3\sqrt[3]{x}}$ is undefined at $x=0$

$y(0) = 0^{5/3} + 2\cdot 0^{2/3} = 0+0 = 0 \to (0,0)$

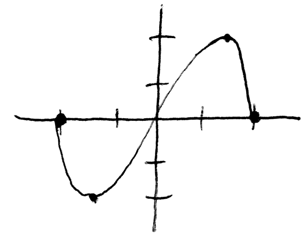
$\frac{5}{3}x^{2/3} + \frac{4}{3x^{1/3}} = 0 \to 5x^{2/3} = -\frac{4}{x^{1/3}} \to 5x^{2/3}\cdot x^{1/3} = -\frac{4}{\cancel{x^{1/3}}}\cdot \cancel{x^{1/3}} \to 5x=-4 \to x = -4/5$

$y(-4/5) = (-4/5)^{5/3} + 2(-4/5)^{2/3} \approx 1.034 \to (-4/5, 1.034)$

$\boxed{\begin{array}{l}\text{Local Min of 0 at } x=0 \text{ (CP, not S)} \\ \text{Local Max of 1.034 at } x=-4/5 \text{ (CP, S)}\end{array}}$

37. $y = x\sqrt{4-x^2}$

$4-x^2 \geq 0 \rightarrow x^2 \leq 4 \rightarrow [-2,2]$ domain with endpoints



$y(2) = 2\sqrt{4-2^2} = 2\cdot 0 = 0 \rightarrow (2,0)$

$y(-2) = 2\sqrt{4-(-2)^2} = 2\cdot 0 = 0 \rightarrow (-2,0)$

$y = x(4-x^2)^{1/2}$

$y' = x \cdot \cancel{\tfrac{1}{2}}(4-x^2)^{-1/2} \cdot -\cancel{2}x + (4-x^2)^{1/2}\cdot 1 = \dfrac{-x^2}{\sqrt{4-x^2}} + \dfrac{\sqrt{4-x^2}}{1}\cdot\dfrac{\sqrt{4-x^2}}{\sqrt{4-x^2}} = \dfrac{-x^2+4-x^2}{\sqrt{4-x^2}} = \dfrac{-2x^2+4}{\sqrt{4-x^2}}$

y' = undefined at $x = \pm 2$ (already got those points above)

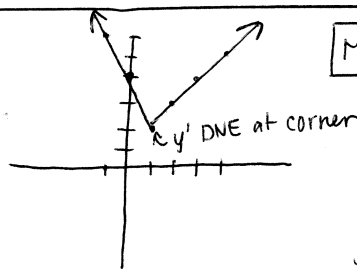
$y' = 0$ when $-2x^2+4 = 0 \rightarrow 2x^2 = 4 \rightarrow x^2 = 2 \rightarrow x = \pm\sqrt{2}$

$y(\sqrt{2}) = \sqrt{2}\cdot\sqrt{4-\cancel{\sqrt{2}}^{\cancel{2}}} = \sqrt{2}\cdot\sqrt{4-2} = \sqrt{2}\cdot\sqrt{2} = 2 \rightarrow (\sqrt{2}, 2)$

$y(-\sqrt{2}) = -\sqrt{2}\cdot\sqrt{4-(-\sqrt{2})^2} = -\sqrt{2}\cdot\sqrt{4-2} = -\sqrt{2}\cdot\sqrt{2} = -2 \rightarrow (-\sqrt{2}, -2)$

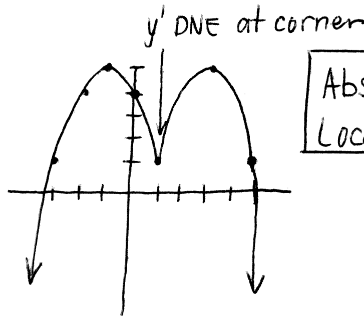
$(2,0) \rightarrow$ Local min of 0 at $x=2$ (CP, not S)
$(-2,0) \rightarrow$ Local max of 0 at $x=-2$ (CP, not S)
$(\sqrt{2},2) \rightarrow$ Max of 2 at $x=\sqrt{2}$ (CP, S)
$(-\sqrt{2},-2) \rightarrow$ Min of -2 at $x=-\sqrt{2}$ (CP, S)

39. $y = \begin{cases} -2x+4, & x \leq 1 \\ x+1, & x > 1 \end{cases}$



Min of 2 at $x=1$ (CP, not S)

11. $y = \begin{cases} -x^2-2x+4, & x \leq 1 & (1,1) \\ -x^2+6x-4, & x > 1 & (1,1) \end{cases}$



Abs. max of 5 at $x=-1$, $x=3$ (CP, S)
Local min of 1 at $x=1$ (CP, not S)

13. $V(x) = x(10-2x)(16-2x)$, $0 < x < 5 \rightarrow$ no endpoints

a) $V(x) = (10x-2x^2)(16-2x) = 160x - 20x^2 - 32x^2 + 4x^3 = 4x^3 - 52x^2 + 160x$

$V'(x) = 12x^2 - 104x + 160 = 0$, never undefined

$3x^2 - 26x + 40 = 0$

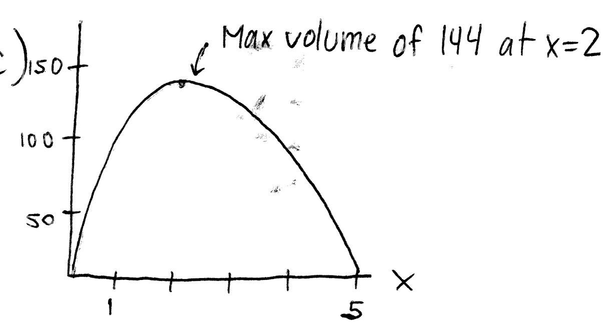
$3x^2 - 20x - 6x + 40 = 0$

$x(3x-20) - 2(3x-20) = 0$

$(x-2)(3x-20) = 0$

$x = 2$, $x = \frac{20}{3} = 6\frac{2}{3} \rightarrow$ not in domain

c)



$V(2) = 2(10-4)(16-4) = 2\cdot 6\cdot 12 = 144$

Max value of 144 at $x = 2$ (CP, S).

b) This result means the maximum volume of the box is 144 cubic units when squares of 2 units in length are removed from each corner and the box is folded up.

53. $f(x) = ax^3 + bx^2 + cx + d$

a) $f'(x) = 3ax^2 + 2bx + c = 0 \rightarrow$ solve w/ quadratic formula

Discriminant $= b^2 - 4ac = (2b)^2 - 4(3a)(c) = 4b^2 - 12ac$

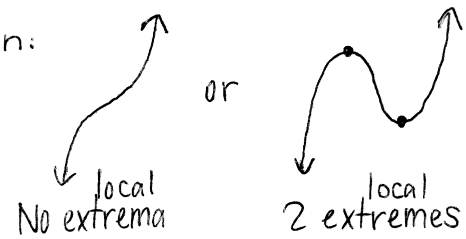
If $4b^2 - 12ac$ is $\rightarrow$ positive $\rightarrow$ 2 solutions $\rightarrow$ 2 CP

zero $\rightarrow$ 1 solution $\rightarrow$ 1 CP

negative $\rightarrow$ no solutions $\rightarrow$ 0 CP

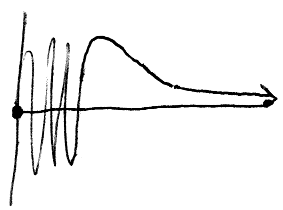
Examples in book

b) Cubic function:



55. $f(x) = \begin{cases} \sin\left(\frac{1}{x}\right), & x > 0 \\ 0, & x = 0 \end{cases}$

a)



There are many x-values where the y-value is 1 or -1, so a y-value of 0 is neither a local min nor local max.

b) $f(x) = \cos\left(\frac{1}{x}\right)$ would work in a similar way.