Section 5.2 : 1–49 e.o.o., 50

1. $f(x) = x^2+2x-1$ on $[0,1]$

f is continuous on $[0,1]$ and differentiable on $(0,1)$, so MVT is satisfied.

Avg. ROC $= \dfrac{f(1)-f(0)}{1-0} = \dfrac{(1^2+2\cdot1-1)-(0+0-1)}{1-0} = \dfrac{2--1}{1-0} = \dfrac{3}{1} = 3$

$f'(x) = 2x+2 \rightarrow 2x+2 = 3$ when $2x = 1 \rightarrow \boxed{c = 1/2}$

5. $f(x) = \sin^{-1}x$ on $[-1,1]$

f is continuous on $[-1,1]$ and differentiable on $(-1,1)$, so MVT is satisfied.

Avg. ROC $= \dfrac{f(1)-f(-1)}{1--1} = \dfrac{\sin^{-1}1-\sin^{-1}(-1)}{2} = \dfrac{\pi/2 - -\pi/2}{2} = \dfrac{\pi}{2}$

$f'(x) = \dfrac{1}{\sqrt{1-x^2}} \rightarrow \dfrac{1}{\sqrt{1-x^2}} = \dfrac{\pi}{2} \rightarrow \sqrt{1-x^2} = \dfrac{2}{\pi} \rightarrow 1-x^2 = \dfrac{4}{\pi^2} \rightarrow x^2 = 1-\dfrac{4}{\pi^2} \rightarrow \boxed{c = \pm\sqrt{1-4/\pi^2} \quad c \approx \pm 0.771}$

7. $f(x) = x + \frac{1}{x} = x + x^{-1}, \quad 0.5 \le x \le 2$

a) $f(1/2) = \frac{1}{2} + \frac{1}{1/2} = \frac{1}{2} + 2 = \frac{5}{2} \rightarrow A\,(1/2, 5/2)$

$f(2) = 2 + \frac{1}{2} = \frac{5}{2} \rightarrow B\,(2, 5/2)$

secant slope : $\dfrac{f(2)-f(1/2)}{2-1/2} = \dfrac{5/2-5/2}{2-\frac{1}{2}} = \dfrac{0}{3/2} = 0$

secant line : $y - \frac{5}{2} = 0(x-1/2) \rightarrow y - \frac{5}{2} = 0 \rightarrow \boxed{y = \frac{5}{2}}$

b) $f'(x) = 1 - x^{-2} = 1 - \frac{1}{x^2}$

$1 - \frac{1}{x^2} = 0 \rightarrow \frac{-1}{x^2} = -1 \rightarrow \frac{1}{x^2} = 1 \rightarrow x^2 = 1 \rightarrow x = \pm 1$

$x=1$ is in interval ✓

$x=-1$ is not in interval ×

$f(1) = 1 + \frac{1}{1} = 1+1 = 2 \rightarrow (1,2)$

$y-2 = 0(x-1) \rightarrow y-2 = 0 \rightarrow \boxed{y=2}$

3. (0 hr, 0 sea miles) (24 hr, 184 sea miles)

Avg. ROC $= \dfrac{\Delta s}{\Delta t} = \dfrac{184-0}{24-0} = \dfrac{184}{24} = 7.667$ sea miles per hour $= 7.667$ knots

If the average speed was 7.667 knots, then the instantaneous speed must have been 7.667 knots at least once, which exceeds 7.5 knots.

17. $h(x) = \frac{2}{x} = 2x^{-1}$

$h'(x) = -2x^{-2} = \frac{-2}{x^2} \neq 0$, but is undefined when $x=0$

However $x=0$ is not in the domain of f bc $\frac{2}{x} = \frac{2}{0} \to$ DNE

$\boxed{\begin{array}{l} a)\ \text{None} \\ b)\ \text{None} \\ c)\ (-\infty,0) \cup (0,\infty) \end{array}}$

$h'(x)$ $\underset{x=0}{- - - \text{DNE} - - -}$ Always decreasing bc $\frac{-2}{x^2} = \frac{-2}{+} = -$ for any x

21. $y = 4 - \sqrt{x+2} = -\sqrt{x+2} + 4$

Domain of y: $x+2 \geq 0 \to x \geq -2 \to [-2,\infty)$

Endpoint: $y(-2) = -\sqrt{-2+2} + 4 = -\sqrt{0} + 4 = 0 + 4 = 4 \to (-2,4)$

$y' = -\frac{1}{2}(x+2)^{-1/2} = \frac{-1}{2\sqrt{x+2}} \neq 0$ & y' DNE when $x=-2$ (already got that point above)

y' $\underset{x=-2}{\text{DNE}} - - - - - -$

$\boxed{\begin{array}{l} a)\ \text{Max of } 4 \text{ at } x=-2 \\ b)\ \text{None} \\ c)\ (-2,\infty) \end{array}}$

25. $h(x) = \frac{-x}{x^2+4}$

Domain: $(-\infty,\infty) \to$ no endpoints

$h'(x) = \frac{(x^2+4)(-1) - (-x)(2x)}{(x^2+4)^2} = \frac{-x^2-4+2x^2}{(x^2+4)^2} = \frac{x^2-4}{(x^2+4)^2} = 0$ when $x^2-4=0 \to x^2=4 \to x=\pm 2$

$h'(x)$ DNE when $(x^2+4)^2=0 \to$ never

$h'(x)$ $+ + + \underset{x=-2}{\overset{\text{Max}}{0}} - - - \underset{x=2}{\overset{\text{Min}}{0}} + + +$ $\quad \cap \quad \cup$

$h'(0) = \frac{-4}{4^2} = \frac{-1}{4} = -$

$h'(3) = \frac{5}{169} = +$

$h(2) = \frac{-2}{2^2+4} = \frac{-2}{4+4} = \frac{-2}{8} = \frac{-1}{4} \to (2,-1/4)$

$h(-2) = \frac{2}{4+4} = \frac{2}{8} = \frac{1}{4} \to (-2,1/4)$

$\boxed{\begin{array}{l} a)\ \text{Min of } -1/4 \text{ at } x=2 \\ \quad\ \text{Max of } 1/4 \text{ at } x=-2 \\ b)\ (-\infty,-2) \cup (2,\infty) \\ c)\ (-2,2) \end{array}}$

29. $f'(x) = x \to f(x) = \boxed{\frac{x^2}{2} + C}$

33. $f'(x) = e^x \to f(x) = \boxed{e^x + C}$

37. $f'(x) = \frac{1}{x+2}$, $P(-1,3)$

$f(x) = \ln(x+2) + C$

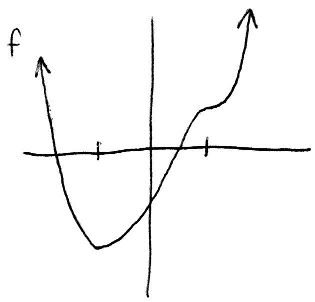
$3 = \ln(-1+2) + C$

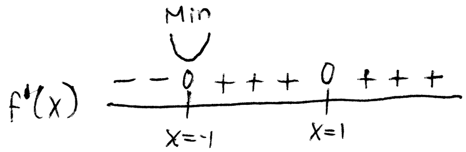
$3 = \ln(1) + C$

$3 = 0 + C$

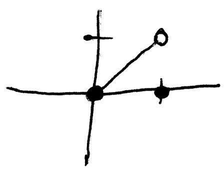
$3 = C \to f(x) = \boxed{\ln(x+2) + 3}$

41. $f'(-1)=0$, $f'(1)=0$, $f'(x)>0$ on $(-1,1)$, $f'(x)<0$ on $(-\infty,-1)$, $f'(x)>0$ on $(1,\infty)$





5. $f(x)=\begin{cases} x, & 0\le x<1 \\ 0, & x=1 \end{cases}$



The function is not continuous on $[0,1]$ bc there is a point discontinuity at $x=1$, so MVT doesn't apply.

19. $f(x)=x+\ln(x+1)$, $0\le x\le 3$

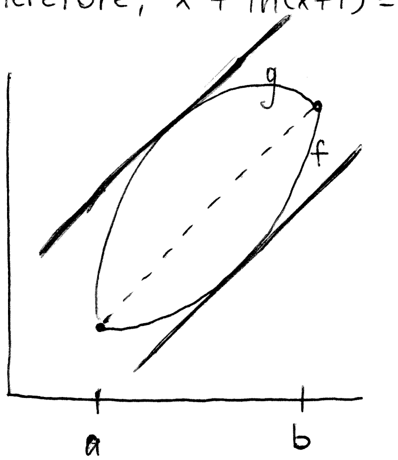
$f(0)=0+\ln(0+1)=0+\ln 1=0+0=0$, so $x=0$ is a solution to the equation.

$f'(x)=1+\frac{1}{x+1}$

$1+\frac{1}{x+1}=0 \rightarrow \frac{1}{x+1}=-1 \rightarrow 1=-x-1 \rightarrow -x=2 \rightarrow x=-2$, but not in interval $0\le x\le 3$

Therefore, $x+\ln(x+1)=0$ has one solution $(x=0)$ on $0\le x\le 3$.

50.



$$\frac{f(b)-f(a)}{b-a}=\frac{g(b)-g(a)}{b-a}$$

The average rate of change ($\Delta y/\Delta x$) is the same for f and g. By Mean Value Theorem, there is at least on point on f on (a,b) and one point on g on (a,b) where the instantaneous slope = the average rate of change. Since the average rates of change are the same for f and g, then the tangent slopes are also the same and the tangent lines are parallel.