Section 5.3: 1–53 odd

1. $y = x^2 - x - 1$

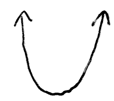
$y' = 2x - 1$

$2x - 1 = 0$ at $x = 1/2$

$y(1/2) = \frac{1}{4} - \frac{1}{2} - 1 = \frac{1}{4} - \frac{2}{4} - \frac{4}{4} = \frac{-5}{4} \rightarrow \boxed{(1/2, -5/4) \rightarrow \text{Abs. Min (CP, S)}}$

y': $- \; - \; 0 \; + \; + \; +$ $2x-1$

$x = 1/2$ Min



3. $y = 2x^4 - 4x^2 + 1$

$y' = 8x^3 - 8x$

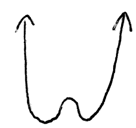
$8x^3 - 8x = 0$

$8x(x^2 - 1) = 0$

$x = 0, x = 1, x = -1$

y': $- \; - \; 0 \; + \; + \; 0 \; - \; - \; 0 \; + \; +$ $8x^3 - 8x$

$x=-1$ Min, $x=0$ Max, $x=1$ Min



$y(-1) = 2 - 4 + 1 = -1 \rightarrow$ $(-1,-1) \rightarrow$ Abs. Min (CP, S)

$y(0) = 0 - 0 + 1 = 1 \rightarrow$ $(0,1) \rightarrow$ Local Max (CP, S)

$y(1) = 2 - 4 + 1 = -1 \rightarrow$ $(1,-1) \rightarrow$ Abs. Min (CP, S)

5. $y = x\sqrt{8-x^2} = x(8-x^2)^{1/2}$

$y' = x \cdot \cancel{\frac{1}{2}}(8-x^2)^{-1/2} \cdot -\cancel{2}x + (8-x^2)^{1/2} \cdot 1$

$y' = \frac{-x^2}{\sqrt{8-x^2}} + \frac{\sqrt{8-x^2}}{1} \cdot \frac{\sqrt{8-x^2}}{\sqrt{8-x^2}} = \frac{-x^2 + 8 - x^2}{\sqrt{8-x^2}} = \frac{8-2x^2}{\sqrt{8-x^2}} = 0$ when $8 - 2x^2 = 0$

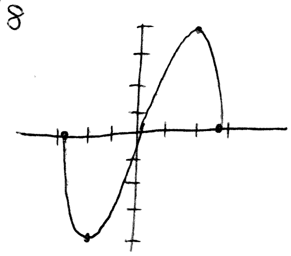
$2x^2 = 8$

$x^2 = 4 \rightarrow x = 2, x = -2$

y' DNE when $\sqrt{8-x^2} = 0 \rightarrow 8 - x^2 = 0 \rightarrow x^2 = 8 \rightarrow x = \sqrt{8}, x = -\sqrt{8}$

y': DNE $- \; - \; 0 \; + \; + \; + \; 0 \; - \; -$ DNE $\frac{8-2x^2}{\sqrt{8-x^2}}$

$x=-\sqrt{8}$ Max, $x=-2$ Min, $x=2$ Max, $x=\sqrt{8}$ Min



$y(2) = 2\sqrt{8-4} = 2 \cdot \sqrt{4} = 2 \cdot 2 = 4 \rightarrow$ $(2,4) \rightarrow$ Abs. Max (CP, S)

$y(-2) = -2\sqrt{8-4} = -2 \cdot \sqrt{4} = -2 \cdot 2 = -4 \rightarrow$ $(-2,-4) \rightarrow$ Abs. Min (CP, S)

$y(\sqrt{8}) = \sqrt{8} \cdot \sqrt{8-8} = \sqrt{8} \cdot 0 = 0 \rightarrow$ $(\sqrt{8}, 0) \rightarrow$ Local Min (End)

$y(-\sqrt{8}) = -\sqrt{8} \cdot \sqrt{8-8} = -\sqrt{8} \cdot 0 = 0 \rightarrow$ $(-\sqrt{8}, 0) \rightarrow$ Local Max (End)

7. $y = 4x^3 + 21x^2 + 36x - 20$

$y' = 12x^2 + 42x + 36$

$y'' = 24x + 42 = 0$ when $x = \frac{-42}{24} = \frac{-7}{4}$

y'': $- \; - \; - \; 0 \; + \; + \; +$ $24x + 42$

$x = \frac{-7}{4}$

CC down: $(-\infty, -7/4)$

CC up: $(-7/4, \infty)$

9. $y = 2x^{1/5} + 3$

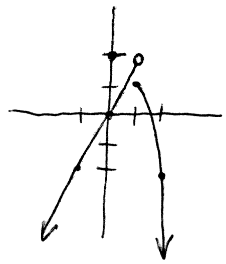
$y' = \frac{2}{5}x^{-4/5}$

$y'' = -\frac{8}{25}x^{-9/5} = \frac{-8}{25x^{9/5}} \neq 0$, but y'' DNE at $x = 0$

$y'' \quad \underset{x=0}{+\ +\ +\ \text{DNE}\ -\ -\ -} \quad \frac{-8}{25x^{9/5}}$

CC up: $\boxed{(-\infty, 0)}$
CC down: $\boxed{(0, \infty)}$

11. $y = \begin{cases} 2x, & x < 1 \\ 2 - x^2, & x \geq 1 \end{cases}$



CC up: $\boxed{\text{None}}$
CC down: $\boxed{(1, \infty)}$

13. $y = xe^x$

$y' = xe^x + e^x \cdot 1 = xe^x + e^x$

$y'' = xe^x + e^x \cdot 1 + e^x = xe^x + 2e^x = e^x(x+2) = 0 \rightarrow e^x \neq 0,\ x+2 = 0$ when $x = -2$

$y'' \quad \underset{x=-2}{-\ -\ 0\ +\ +\ +} \quad e^x(x+2)$

$y(-2) = -2 \cdot e^{-2} = \frac{-2}{e^2} \rightarrow$ POI: $\boxed{(-2, -2/e^2)}$

15. $y = \tan^{-1}x$

$y' = \frac{1}{1+x^2} = (1+x^2)^{-1}$

$y'' = -1(1+x^2)^{-2} \cdot 2x = \frac{-2x}{(1+x^2)^2} = 0$ when $x = 0$

$y'' \quad \underset{x=0}{+\ +\ +\ 0\ -\ -\ -} \quad \frac{-2x}{(1+x^2)^2}$

$y(0) = \tan^{-1}(0) = 0 \rightarrow$ POI: $\boxed{(0, 0)}$

17. $y = x^{1/3}(x-4) = x^{4/3} - 4x^{1/3}$

$y' = \frac{4}{3}x^{1/3} - \frac{4}{3}x^{-2/3}$

$y'' = \frac{4}{9}x^{-2/3} + \frac{8}{9}x^{-5/3} = \frac{4}{9x^{2/3}} + \frac{8}{9x^{5/3}}$ DNE when $x = 0$

$\frac{4}{9x^{2/3}} + \frac{8}{9x^{5/3}} = 0 \rightarrow \frac{4}{\cancel{9}x^{2/3}} = \frac{-8}{\cancel{9}x^{5/3}} \rightarrow \frac{4}{x^{2/3}} \times \frac{-8}{x^{5/3}} \rightarrow \frac{4x^{5/3}}{x^{2/3}} = \frac{-8\cancel{x^{2/3}}}{\cancel{x^{2/3}}}$

$4x = -8 \rightarrow x = -2$

$y'' \quad \underset{x=-2 \qquad\qquad x=0}{+\ +\ 0\ -\ -\ \text{DNE}\ +\ +} \quad \frac{4}{9x^{2/3}} + \frac{8}{9x^{5/3}}$

$y(0) = 0(0-4) = 0(-4) = 0 \rightarrow \boxed{(0, 0)}$

$y(-2) = (-2)^{4/3} - 4(-2)^{1/3} = \underbrace{\sqrt[3]{16}}_{\sqrt[3]{8} \cdot \sqrt[3]{2}} + 4\sqrt[3]{2} = 2\sqrt[3]{2} + 4\sqrt[3]{2} = 6\sqrt[3]{2} \rightarrow \boxed{(-2, 6\sqrt[3]{2})}$

19. $y=\dfrac{x^3-2x^2+x-1}{x-2}$

$y'=\dfrac{(x-2)(3x^2-4x+1)-(x^3-2x^2+x-1)(1)}{(x-2)^2}$

$y'=\dfrac{3x^3-4x^2+\cancel{x}-6x^2+8x-2-x^3+2x^2-\cancel{x}+1}{(x-2)^2}=\dfrac{2x^3-8x^2+8x-1}{(x-2)^2}$

$y''=\dfrac{(x-2)^{\cancel{2}}(6x^2-16x+8)-(2x^3-8x^2+8x-1)\cdot 2\cancel{(x-2)}}{(x-2)^{\cancel{4}\,3}}$

$y''=\dfrac{6x^3-\cancel{16x^2}+8x-12x^2+32x-16-4x^3+\cancel{16x^2}-16x+2}{(x-2)^3}=\dfrac{2x^3-12x^2+24x-14}{(x-2)^3}$

$y''=0$ when $2x^3-12x^2+24x-14=0 \rightarrow$ on graph: $x=1$

$y(1)=\dfrac{1-2+\cancel{1}-\cancel{1}}{1-2}=\dfrac{-1}{-1}=1 \rightarrow$ POI: $\boxed{(1,1)}$

21. f': zero: $x=-1,\ x=1$
pos: $(-\infty,-1)\cup(1,\infty)$
neg: $(-1,1)$

f'': zero: $x=0$
pos: $(0,\infty)$
neg: $(-\infty,0)$

23. a) Inc when $f'(x)>0$: $\boxed{(-\infty,-2)\cup(0,2)}$
b) Dec when $f'(x)<0$: $\boxed{(-2,0)\cup(2,\infty)}$
c) Local max when $f'(x)$ changes from + to − : $x=-2,\ x=2$
Local min when $f'(x)$ changes from − to + : $x=0$

25. $x(t)=t^2-4t+3$
$v(t)=2t-4$
$a(t)=2$

$v(t)$: − − 0 + + + ($2t-4$), $t=0$, $t=2$

$x(0)=0^2-4\cdot 0+3=3$, so the particle starts at 3.
$2t-4=0$ at $t=2$, so the velocity is 0 at $t=2$.
The particle moves left during the first 2 seconds, then right after that.

27. $x(t) = t^3 - 3t + 3$
$v(t) = 3t^2 - 3$
$a(t) = 6t$

$v(t)\ \underset{t=0}{|}\ - \ - \underset{t=1}{\overset{0}{|}}\ + \ +$

$x(0) = 0^3 - 3\cdot 0 + 3$, so the particle starts at 3.
$3t^2 - 3 = 0 \to 3t^2 = 3 \to t^2 = 1$ at $t = 1$ ($t = -1$ doesn't make sense), so the velocity is 0 at $t = 1$. The particle moves left during the first second, then right after that.

29. a) $v = 0$ when slope $= 0 \to t = 2.2, 6, 9.8$
b) $a = 0$ at point of inflection $\to t = 4, 8, 12$

31. a) $y = \dfrac{12{,}655.179}{1 + 12.871e^{-0.033x}}$
b) Graph scatterplot & curve on calc
c) Year 2000 = 180 years after 1820 $\to y(180) = 12{,}240.509$ thousand
$= 12{,}240{,}509$ people
d) Population grew the fastest when the slope of the curve was the steepest, which is about $x = 80$, so around the year 1900. This point on the curve is the point of inflection.
e) As time goes on, the population won't grow much any more, leveling out at approximately 12,655,179 because

$$\lim_{x\to\infty} \frac{12{,}655.179}{1 + \frac{12.871}{e^{0.33x}}} = \frac{12{,}655.179}{1 + \frac{12.871}{\infty}} = \frac{12{,}655.179}{1 + 0} = 12{,}655.179 \text{ thousand}$$
$= 12{,}655{,}179$ people

33. $y = 3x - x^3 + 5$
$y' = 3 - 3x^2 \to y' = 0$ when $3 - 3x^2 = 0 \to 3x^2 = 3 \to x^2 = 1 \to x = 1, x = -1$
$y(1) = 3 - 1 + 5 = 7 \to (1,7)$
$y(-1) = -3 + 1 + 5 = 3 \to (-1,3)$
$y'' = -6x$
At $x = 1$, $y''(1) = -6(1) = -6 \to$ concave down, so $\boxed{\text{max at } (1,7)}$
At $x = -1$, $y''(-1) = -6(-1) = 6 \to$ concave up, so $\boxed{\text{min at } (-1,3)}$

35. $y = x^3 + 3x^2 - 2$
$y' = 3x^2 + 6x \to y' = 0$ when $3x^2 + 6x = 0 \to 3x(x+2) = 0 \to x = 0, x = -2$
$y(0) = 0 + 0 - 2 = -2 \to (0,-2)$
$y(-2) = -8 + 12 - 2 = 2 \to (-2, 2)$
$y'' = 6x + 6$
At $x = 0$, $y''(0) = 6(0) + 6 = 6 \to$ concave up, so $\boxed{\text{min at } (0,-2)}$
At $x = -2$, $y''(-2) = 6(-2) + 6 = -6 \to$ concave down, so $\boxed{\text{max at } (-2,2)}$

37. $y = xe^x$

$y' = xe^x + e^x \cdot 1 = xe^x + e^x = e^x(x+1) = 0 \rightarrow \quad e^x \neq 0,\ x+1 = 0$ when $x = -1$

$y(-1) = -1 \cdot e^{-1} = \frac{-1}{e} \rightarrow (-1, -1/e)$

$y'' = xe^x + e^x \cdot 1 + e^x = xe^x + 2e^x = e^x(x+2)$

$y''(-1) = e^{-1}(-1+2) = \frac{1}{e} \cdot 1 = \frac{1}{e} = + \rightarrow$ concave up, so $\boxed{\text{min at } (-1, -1/e)}$

39. $y' = (x-1)^2(x-2)$

$y' = 0$ at $x = 1,\ x = 2$

a) $\boxed{\text{None}}$
b) $\boxed{x=2}$

y': $-\ -\ 0\ -\ -\ 0\ +\ +$ $\quad (x-1)^2(x-2)$

$x=1$ Neither, $x=2$ Min

$y'' = (x-1)^2 \cdot 1 + (x-2) \cdot 2(x-1) = (x-1)^2 + 2(x-2)(x-1)$

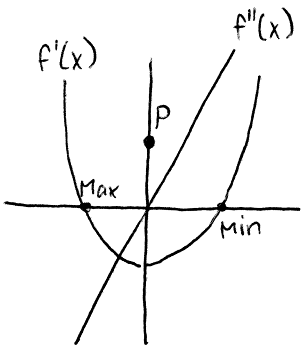
$y'' = (x-1)[(x-1) + 2(x-2)] = (x-1)(x-1+2x-4) = (x-1)(3x-5) = 0$ at

c) $\boxed{x=1,\ x=5/3}$

y'': $+\ +\ 0\ -\ -\ 0\ +\ +$ $\quad (x-1)(3x-5)$

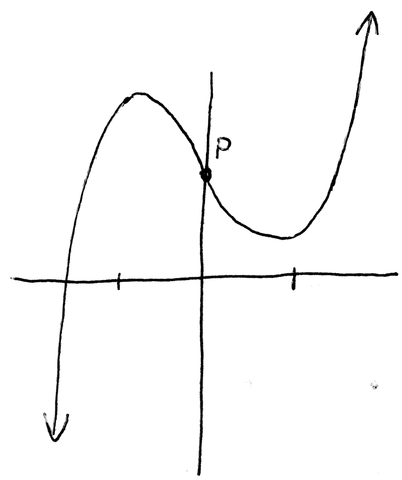
$x=1$, $x=5/3$

41.



$f''(x)$: CC down $(-\infty, 0)$
CC up $(0, \infty)$

$f'(x)$: Inc, then dec, then inc

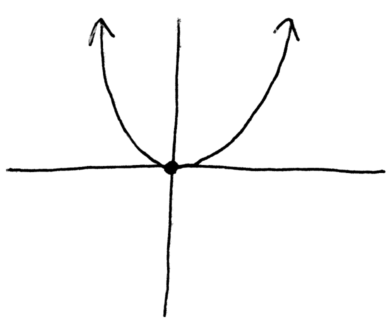


43. Not necessarily. Besides $f'(c) = 0$, the sign of $f'(x)$ must change from $+$ to $-$, or $-$ to $+$, in order for $f(x)$ to have a local min or max.

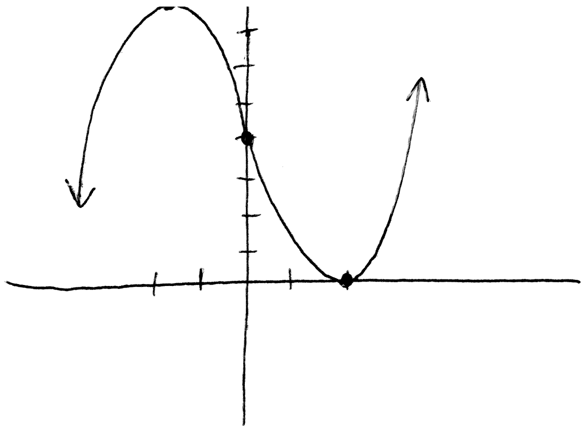
45. Origin

$f'(x) < 0$ for $x < 0$ (Dec)

$f'(x) > 0$ for $x > 0$ (Inc)



47.



	Max		Neither		Min	
Inc, C down		Dec, C down		Dec, C up		Inc, C up
	$x=-2$		$x=0$		$x=2$	

49. a) Inc when f' is $+$: $(0,1) \cup (3,4) \cup (5.5,6)$

b) Dec when f' is $-$: $(1,3) \cup (4,5.5)$

c) $x=0 \to$ Local Min (inc after)

$x=1 \to$ Local Max (inc to dec)

$x=3 \to$ Local Min (dec to inc)

$x=4 \to$ Local Max (inc to dec)

$x=5.5 \to$ Local Min (dec to inc)

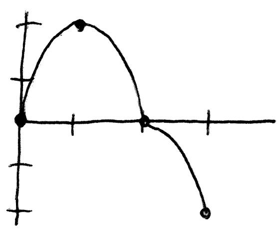
$x=6 \to$ Local Max (inc before)

51. a) $x=1 \to$ Abs. Max (f' changes $+$ to $-$) at $(1,2)$

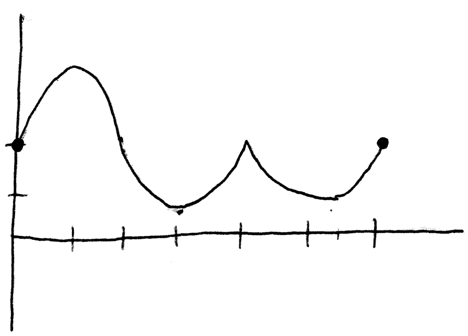
$x=3 \to$ Abs. Min (endpoint) at $(3,-2)$

b) None (f'' always $- \to$ CC down)

c)



53.



Inc, C down	Dec, C down	Dec, C up	Inc, C up	Dec, C up	Inc, C up
$x=0$ to $x=1$	$x=1$ to $x=2$	$x=2$ to $x=3$	$x=3$ to $x=4$	$x=4$ to $x=5.5$	$x=5.5$ to $x=6$