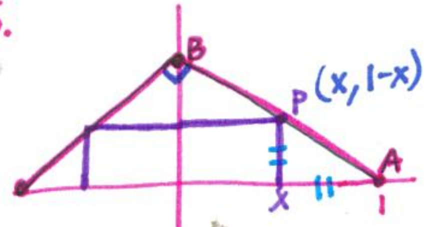
Section 5.4

5.



$\overline{AB}$ is $y=-x+1$

a) $P(x, 1-x)$

b) $V = l \cdot w$

$= (2x)(1-x)$

$\boxed{V = 2x - 2x^2}$

c) $V' = 2 - 4x = 0$

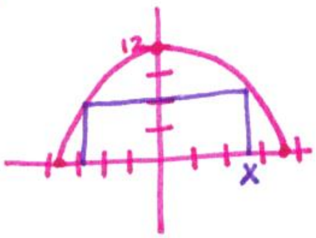
$2 = 4x$

$x = 1/2$

$w = 2x \rightarrow \boxed{w = 1}$

$l = 1-x \rightarrow \boxed{l = 1/2}$

6.



$A = l \cdot w$

$\downarrow$

$A = (2x)(12-x^2)$

$A = 24x - 2x^3$

$A' = 24 - 6x^2 = 0$

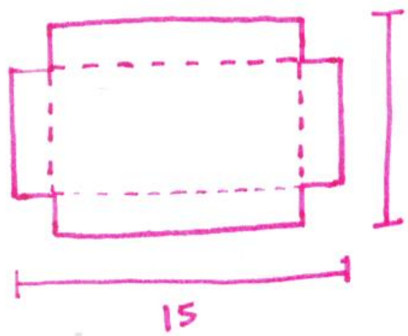
$6x^2 = 24$

$x^2 = 4$

$x = \pm 2 \rightarrow \underline{\underline{2}}$

$\boxed{w = 12 - x^2 \rightarrow 8 \quad l = 2x \rightarrow 4 \quad A_{max} = 8 \cdot 4 = \underline{\underline{32}}}$

7.



$V = l \cdot w \cdot h$

$= (15-2x)(8-2x)x$

$V = 4x^3 - 46x^2 + 120x$

$V' = 12x^2 - 92x + 120$

$\swarrow \quad \searrow$

$\frac{5}{3} \qquad \cancel{6}$ Not in the interval

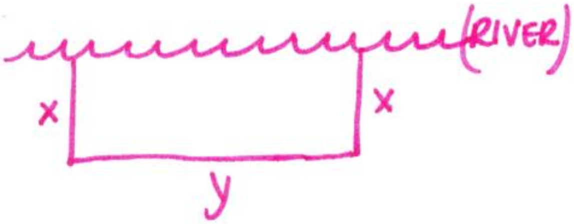
$l = 15 - 2(5/3) = 11.\bar{6}$ in

$w = 8 - 2(5/3) = 4.\bar{6}$ in

$h = 5/3$ in

$V_{max} = 90.741$ in^3

9.



$800 = 2x + y$

$y = 800 - 2x$

$A = x \cdot y$

$= x(800 - 2x)$

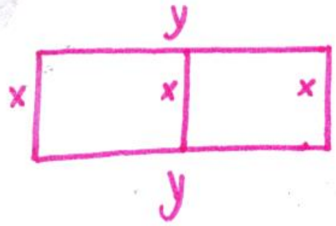
$A = 800x - 2x^2$

$A' = 800 - 4x = 0$

$4x = 800$

$\boxed{x = 200 \text{ m}, \ y = 400 \text{ m}}$

$\boxed{A_{max} = (200)(400) = 80{,}000 \text{ m}^2}$



$$A = x \cdot y = 216$$

$$y = \frac{216}{x}$$

$$P = 2y + 3x$$

$$= 2\left(\frac{216}{x}\right) + 3x$$

$$P = \frac{432}{x} + 3x$$

$$\frac{dP}{dx} = \frac{-432}{x^2} + 3 = 0$$

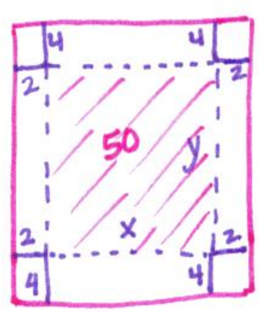
$$3 = \frac{432}{x^2}$$

$$3x^2 = 432$$

$$x^2 = 144$$

$$\boxed{\begin{array}{c} x = 12 \rightarrow y = 18 \\ P = 2(18) + 3(12) = 72\text{ m} \end{array}}$$

13.



$$x \cdot y = 50$$

$$y = \frac{50}{x}$$

overall dim: $l \cdot w$

$$l = x + 4$$

$$w = y + 8 = \frac{50}{x} + 8$$

$$A = (x+4)\left(\frac{50}{x} + 8\right)$$

$$A = 8x + 82 + \frac{200}{x}$$

$$A' = 8 - \frac{200}{x^2} = 0$$

$$8 = \frac{200}{x^2}$$

$$8x^2 = 200$$

$$x^2 = 25$$

$$x = 5$$

$$\boxed{\begin{array}{l} l = 5 + 4 = 9 \\ w = \frac{50}{5} + 8 = 18 \end{array}}$$

$A = \frac{1}{2}ab\sin\theta$ $\quad A' = \frac{1}{2}ab(\cos\theta) = 0$

both are constants!

$\cos\theta = 0$

$\theta = \pi/2,\ 3\pi/2$

$\boxed{\therefore\ \theta = \pi/2 \text{ will maximize the Area of the Triangle.}}$

A': $+$ | $\pi/2$ | $-$ | $3\pi/2$ | $+$

max min

16. $V = 1000 = \pi r^2 h$ $\quad SA = \pi r^2 + 2\pi r h$

$h = \frac{1000}{\pi r^2}$

only one circle $= \pi r^2 + 2\pi r\left(\frac{1000}{\pi r^2}\right)$

$SA = \pi r^2 + \frac{2000}{r}$

$h = \frac{1000}{\pi(6.828)^2}$

$\boxed{h = 6.828 \text{ cm}}$

$\frac{dS}{dr} = 2\pi r - \frac{2000}{r^2} = 0$

$2\pi r = \frac{2000}{r^2}$

$2\pi r^3 = 2000$

$\boxed{r = \sqrt[3]{\frac{1000}{\pi}} = 6.828 \text{ cm}}$

17. $A = 8r^2 + 2\pi r h$ $\quad h = \frac{1000}{\pi r^2}$ $\quad \boxed{h = \frac{1000}{25\pi} = \frac{40}{\pi} = 12.732}$

$A = 8r^2 + 2\pi r\left(\frac{1000}{\pi r^2}\right)$ $\quad \boxed{\text{Ratio: } 1 \text{ to } \frac{8}{\pi}}$

$A = 8r^2 + \frac{2000}{r}$

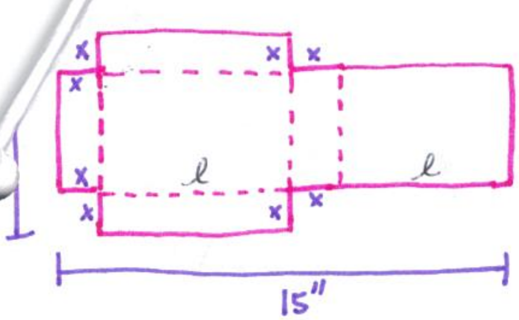
$A' = 16r - \frac{2000}{r^2} = 0$

$16r = \frac{2000}{r^2}$

$16r^3 = 2000$

$r = \sqrt[3]{125}$

$\boxed{r = 5}$



$2l + 2x = 15$

$2(l + x) = 15$

$l + x = \frac{15}{2}$

$l = \frac{15}{2} - x$

$w = 10 - 2x$

$h = x$

$V = \left(\frac{15}{2} - x\right)(10 - 2x)x$

$V = 2x^3 - 25x^2 + 75x$

$V' = 6x^2 - 50x + 75$

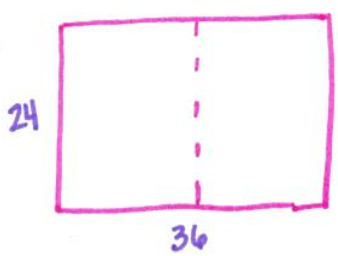
$1.962 \quad \cancel{6.371}$

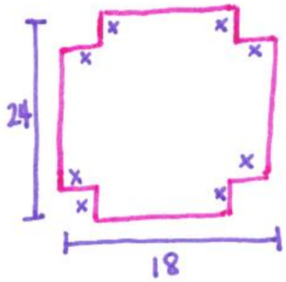
$(0 < x < 5)$

$\boxed{\begin{array}{l} l = \frac{15}{2} - 1.962 = 5.538 \text{ in.} \\ w = 10 - 2(1.962) = 6.076 \text{ in.} \\ h = 1.962 \text{ in.} \end{array}}$

$\boxed{V_{max} = 66.019 \text{ in}^3}$

19.





$V = (l \cdot w \cdot h)2$

$V = [(18 - 2x)(24 - 2x)x]2$

a) $\boxed{V = 8x^3 - 168x^2 + 864x}$

c) $V' = 24x^2 - 336x + 864$

$3.394 \quad \cancel{10.606}$

$\boxed{x = 3.394}$

b) $18 - 2x > 0$

$18 > 2x$

$\boxed{x < 9}$

e) $1120 = 8x^3 - 168x^2 + 864x$

$0 = 8x^3 - 168x^2 + 864x - 1120$

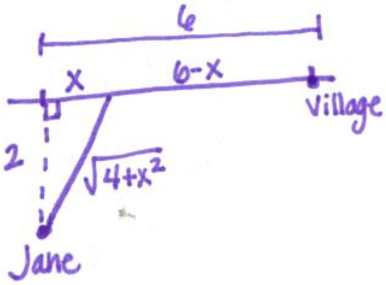
$0 = 8(x^3 - 21x^2 + 108x - 140)$ * use the graph!

$\boxed{x = 2, 5, \cancel{14}}$

Not in the domain

...ction 5.4

20.



$f(x) = \frac{\sqrt{4+x^2}}{2} + \frac{6-x}{5}$

$f(x) = \frac{1}{2}(4+x^2)^{1/2} + \frac{1}{5}(6-x)$

$f'(x) = \frac{1}{4}(4+x^2)^{-1/2}(2x) + \frac{1}{5}(-1)$

$0 = \frac{x}{2(4+x^2)^{1/2}} - \frac{1}{5}$

$\frac{1}{5} = \frac{x}{2(4+x^2)^{1/2}}$

$\left(2(4+x^2)^{1/2}\right)^2 = (5x)^2$

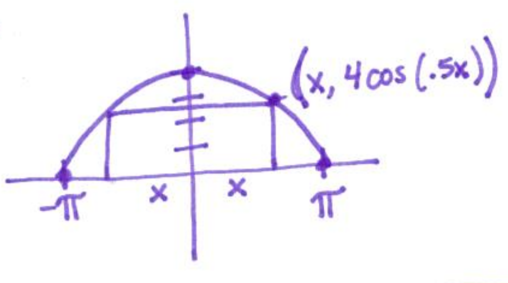
$4(4+x^2) = 25x^2$

$16 + 4x^2 = 25x^2$

$16 = 21x^2$

$x = \sqrt{\frac{16}{21}} = .873$ ∴ Jane should land her boat .873 mi down the shoreline

21.



$A = l \cdot w$

$A = (2x)(4\cos .5x)$

$A = 8x\cos(.5x)$

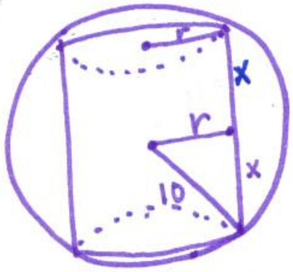
$A' = 8x\left(-\sin .5x \cdot \frac{1}{2}\right) + (\cos .5x) \cdot 8$

$0 = -4x\sin(.5x) + 8\cos .5x$

$4x\sin(.5x) = 8\cos(.5x)$ ← use the graphs & see where they intersect.

$x = 1.721$

$$\boxed{\begin{array}{l} l = 2x = 2(1.72) = 3.442 \\ w = 4\cos .5x = 4\cos(.5 \cdot 1.72) = 2.608 \end{array}}$$



$V=\pi r^2 h$ $\quad h=2x$

$V=\pi r^2\left(2\sqrt{100-r^2}\right)$ $\quad x=\sqrt{100-r^2}$ $\quad r^2=100-x^2$

$V=2\pi r^2(100-r^2)^{1/2}$ $\quad h=2\sqrt{100-r^2}$

*Blue is a MUCH easier path!

$V=\pi(100-x^2)(2x)$

$V=200\pi x-2\pi x^3$

$V'=200\pi-6\pi x^2$

$\downarrow$

$x=\sqrt{100/3}$

$V'=2\pi r^2\left[\frac{1}{2}(100-r^2)^{-1/2}(-2r)\right]+(100-r^2)^{1/2}(4\pi r)$

$=\dfrac{-2\pi r^3}{(100-r^2)^{1/2}}+\dfrac{4\pi r(100-r^2)^{1/2}}{1}\cdot\dfrac{(100-r^2)^{1/2}}{(100-r^2)^{1/2}}$

$=\dfrac{-2\pi r^3+4\pi r(100-r^2)}{(100-r^2)^{1/2}}$

$V'=\dfrac{400\pi r-6\pi r^3}{(100-r^2)^{1/2}}=\dfrac{2\pi r(200-3r^2)}{(100-r^2)^{1/2}}$

$0<r<10$

$200-3r^2=0$

$200=3r^2$

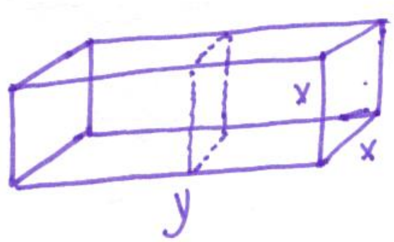
$\boxed{r=\sqrt{\frac{200}{3}}=8.165\text{ cm}}$

$h=2\sqrt{100-(8.165)^2}$

$\boxed{h=11.547\text{ cm}}$

$\boxed{\begin{aligned}V_{max}&=\pi(8.165)^2(11.547)\\&=2418.418\text{ cm}^3\end{aligned}}$

30.



$l+g=108$

$\downarrow$

$y+4x=108$

$y=108-4x$

$V=l\cdot w\cdot h$

$=(108-4x)\cdot x\cdot x$

$V=108x^2-4x^3$

$V'=216x-12x^2$

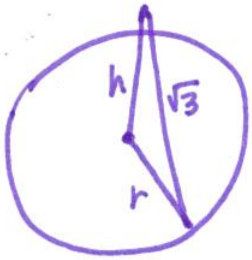
$0=12x(18-x)$

$\downarrow \quad \downarrow$

$0 \quad 18$

$\boxed{\begin{aligned}l&=108-4(18)=36\\w&=18\\h&=18\end{aligned}}$

.



$V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi(3-h^2)h$

$V = \pi h - \frac{\pi}{3}h^3$

$V' = \pi - \frac{\pi}{3} \cdot 3h^2$

$= \pi - \pi h^2$

$0 = \pi(1-h^2)$

$1-h^2 = 0$

$1 = h^2$

$h = 1\text{m}$

$h^2 + r^2 = (\sqrt{3})^2$

$r^2 = 3-h^2$

$r^2 = 3-(1)^2$

$\boxed{r = \sqrt{2}\text{m}}$

$V_{cone} = \frac{\pi}{3}(\sqrt{2})^2(1)$

$= \frac{2\pi}{3}\text{m}^3$

34. $f(x) = x^2 + \frac{a}{x}$

$f'(x) = 2x - \frac{a}{x^2} = 0$

$2x = \frac{a}{x^2}$

$2x^3 = a$

$x = \sqrt[3]{\frac{a}{2}}$

$f'(x)$: $-$ | $+$ at $\sqrt[3]{\frac{a}{2}}$

$\therefore \sqrt[3]{\frac{a}{2}}$ is a MIN & there is no max!

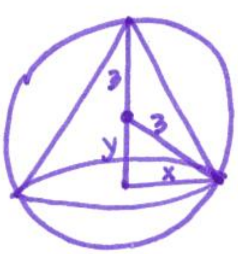
OR

$f''(x) = 2 + \frac{2a}{x^3}$ is always (+) $\therefore$ ccup so $x = \sqrt[3]{\frac{a}{2}}$ must be a MIN!

$= 2 + \frac{2(2x^3)}{x^3}$

$= 6$

36.



$x^2 + y^2 = 3^2$

$x^2 = 9 - y^2$

$V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi x^2 (y+3)$

$= \frac{1}{3}\pi(9-y^2)(y+3)$

$V = \frac{\pi}{3}(27 - 3y^2 + 9y - y^3)$

$V' = \frac{\pi}{3}(-6y + 9 - 3y^2)$

$= \pi(-2y + 3 - y^2)$

$0 = -\pi(y^2 + 2y - 3)$

$= -\pi(y+3)(y-1)$

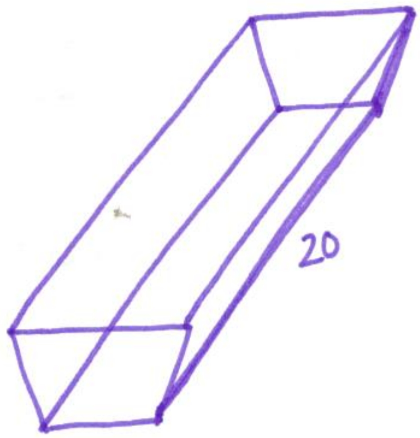
$\cancel{-3}$ 1

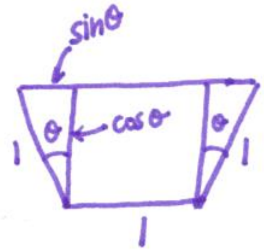
$\boxed{y = 1 \quad h = 4 \quad x = \sqrt{8}}$

$V = \frac{1}{3}\pi(8)(4)$

$\boxed{V = \frac{32\pi}{3}}$

11.





height = $\cos\theta$

small base = 1

large base = $1+2\sin\theta$

$$V = \frac{1}{2}(SB+LB)\cdot h \cdot l$$

$$= \frac{1}{2}(1+1+2\sin\theta)\cdot\cos\theta\cdot 20$$

$$= 10\cos\theta(2+2\sin\theta)$$

$$V = 20\cos\theta(1+\sin\theta)$$

$$V' = 20\cos\theta(\cos\theta) + (1+\sin\theta)(-20\sin\theta)$$

$$= 20\cos^2\theta - 20\sin\theta - 20\sin^2\theta$$

$$= 20(1-\sin^2\theta) - 20\sin\theta - 20\sin^2\theta$$

$$= 20 - 20\sin^2\theta - 20\sin\theta - 20\sin^2\theta$$

$$= -20\left[2\sin^2\theta + \sin\theta - 1\right]$$

$$0 = -20(2\sin\theta - 1)(\sin\theta + 1)$$

$\sin\theta = 1/2$ $\quad$ $\sin\theta = -1$ ← we don't want this to equal −1.

$\boxed{\theta = \pi/6}$ $\quad$ $\cancel{\theta = 3\pi/2}$ out of interval