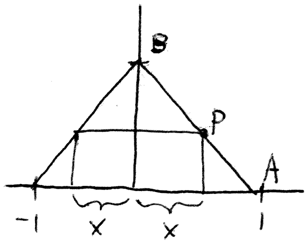
Section 5.4 : 5-10, 13, 15-22, 30, 32, 34, 36, 47

5.



$b = 2x$

$h = -x+1$

a) $\boxed{y = -x+1}$ (equation of line AB)

b) $A(x) = bh$

$A(x) = 2x(-x+1) = \boxed{-2x^2+2x}$

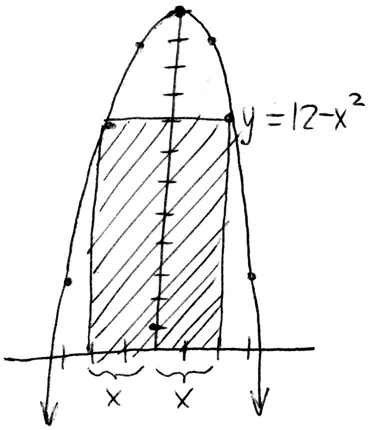
c) $A'(x) = -4x+2 = 0$ when $4x = 2 \rightarrow x = 1/2$

$b = 2x = 2(1/2) = \boxed{1}$

$h = -(\frac{1}{2}) + 1 = \boxed{1/2}$

$A = bh = 1(1/2) = \boxed{1/2}$

6.



$b = 2x, \ h = 12-x^2$

$A = bh = 2x(12-x^2) = 24x - 2x^3$

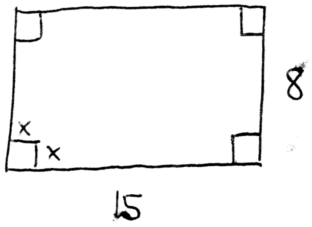
$A' = 24 - 6x^2 = 0$ when $6x^2 = 24 \rightarrow x^2 = 4 \rightarrow x = 2$

$b = 2x = 2 \cdot 2 = \boxed{4}$

$h = 12 - x^2 = 12 - 2^2 = \boxed{8}$

$A = 4 \cdot 8 = \boxed{32}$

7.



$l = 15-2x, \ w = 8-2x, \ h = x$

$V = lwh = (15-2x)(8-2x)x$

$V = (120 - 30x - 16x + 4x^2)x = 120x - 46x^2 + 4x^3$

$V' = 120 - 92x + 12x^2 = 0$

$12x^2 - 92x + 120 = 0$

$$x = \frac{92 \pm \sqrt{92^2 - 4(12)(120)}}{2(12)} = \frac{92 \pm \sqrt{2704}}{24} \begin{array}{l} \nearrow 6 \text{ (can't use bc } 8 - 2 \cdot 6 = -4) \\ \searrow 1.\overline{6} = \frac{5}{3} \end{array}$$

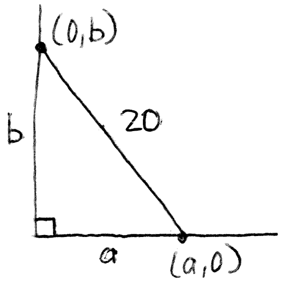
$l = 15 - 2x = 15 - 2(\frac{5}{3}) = \frac{45}{3} - \frac{10}{3} = \boxed{\frac{35}{3} \text{ in}}$

$w = 8 - 2x = 8 - 2 \cdot \frac{5}{3} = \frac{24}{3} - \frac{10}{3} = \boxed{\frac{14}{3} \text{ in}}$

$h = \boxed{\frac{5}{3} \text{ in}}$

$V = lwh = \frac{35}{3} \cdot \frac{14}{3} \cdot \frac{5}{3} = \frac{2450}{27} \text{ in}^3 \approx \boxed{90.741 \text{ in}^3}$

8.



base = a

height = b

$A = \frac{1}{2}bh = \frac{1}{2}ab$

$a^2+b^2=20^2 \to a^2+b^2=400 \to b^2=400-a^2 \to b=\sqrt{400-a^2}$

$$A = \frac{1}{2}a\sqrt{400-a^2} = \frac{1}{2}a(400-a^2)^{1/2}$$

$$A' = \frac{1}{2}a\cdot\cancel{\frac{1}{2}}(400-a^2)^{-1/2}\cdot -\cancel{2}a + (400-a^2)^{1/2}\cdot\frac{1}{2}$$

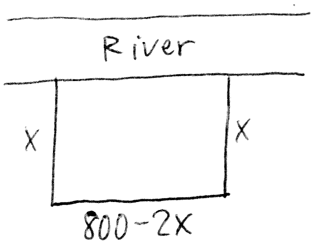
$$A' = \frac{-a^2}{2\sqrt{400-a^2}} + \frac{\sqrt{400-a^2}}{2}\cdot\frac{\sqrt{400-a^2}}{\sqrt{400-a^2}}$$

$$A' = \frac{-a^2+(400-a^2)}{2\sqrt{400-a^2}} = \frac{-a^2+400-a^2}{2\sqrt{400-a^2}} = \frac{400-2a^2}{2\sqrt{400-a^2}} = \frac{200-a^2}{\sqrt{400-a^2}} = 0$$

when $200-a^2=0 \to a^2=200 \to a=\sqrt{200}$ — $\boxed{a=b}$

$b=\sqrt{400-a^2} = \sqrt{400-\cancel{\sqrt{200}}^{\cancel{2}}} = \sqrt{400-200} = \sqrt{200}$

9.



$A = lw = (800-2x)(x) = 800x-2x^2$

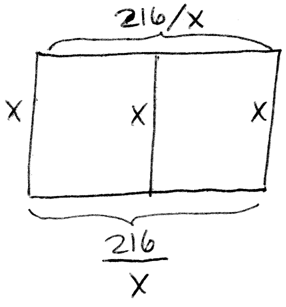
$A' = 800-4x = 0$ when $4x=800 \to x=200$

$w = \boxed{200\text{ m}}$

$l = 800-2x = 800-2(200) = 800-400 = \boxed{400\text{ m}}$

$A = lw = 400\cdot 200 = \boxed{80{,}000\text{ m}^2}$

10. $A = 216\text{ m}^2$



$P = x+x+x+\frac{216}{x}+\frac{216}{x}$

$P = 3x+\frac{432}{x} = 3x+432x^{-1}$

$P' = 3-432x^{-2} = 3-\frac{432}{x^2} = 0$

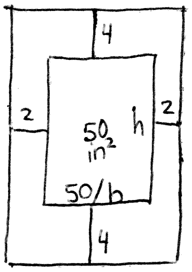
$3 = \frac{432}{x^2} \to 3x^2=432 \to x^2=144 \to x=12\text{ m}$

$w = \boxed{12\text{ m}}$

$l = \frac{216}{x} = \frac{216}{12} = \boxed{18\text{ m}}$

$P = 3(12)+2(18) = 36+36 = \boxed{72\text{ m}}$

13.



height = h+8
width = $\frac{50}{h} + 4$

$A = (h+8)(\frac{50}{h}+4)$

$A = 50 + 4h + \frac{400}{h} + 32$

$A = 4h + 400h^{-1} + 82$

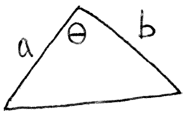
$A' = 4 - 400h^{-2} = 4 - \frac{400}{h^2} = 0$ when $\frac{400}{h^2} = 4$

$4h^2 = 400 \rightarrow h^2 = 100 \rightarrow h = 10$ in

$w = \frac{50}{h} = \frac{50}{10} = 5$ in $\leftarrow$ dimensions of printed area

Overall dimensions: height $= 10$ in $+ 4$ in $+ 4$ in $=$ $\boxed{18 \text{ in}}$ (with margins)
width $= 5$ in $+ 2$ in $+ 2$ in $=$ $\boxed{9 \text{ in}}$

15.



$A = \frac{1}{2}ab\sin\theta$

a, b are side lengths, so constant numbers.

$A' = \frac{1}{2}ab\cos\theta = 0$

$\cos\theta = 0$ at $\boxed{\theta = \pi/2}$

16. $V = 1000 \text{ cm}^3$

$\pi r^2 h = 1000 \rightarrow h = \frac{1000}{\pi r^2}$

$S = \pi r^2 + 2\pi r h$

↑ not $2\pi r^2$ here bc the can does not have a lid

$S = \pi r^2 + 2\cancel{\pi}\cancel{r}\cdot\frac{1000}{\cancel{\pi} r^{\cancel{2}}} = \pi r^2 + \frac{2000}{r} = \pi r^2 + 2000r^{-1}$

$S' = 2\pi r - 2000r^{-2} = 2\pi r - \frac{2000}{r^2} = 0 \rightarrow 2\pi r = \frac{2000}{r^2}$

$2\pi r^3 = 2000 \rightarrow \pi r^3 = 1000 \rightarrow r^3 = \frac{1000}{\pi} \rightarrow r = \sqrt[3]{\frac{1000}{\pi}} = \boxed{6.828 \text{ cm}}$

$h = \frac{1000}{\pi r^2} = \frac{1000}{\pi \cdot 6.828^2} = \boxed{6.828 \text{ cm}}$

17. $A = 8r^2 + 2\pi r h$

$V = 1000 \rightarrow \pi r^2 h = 1000 \rightarrow h = \frac{1000}{\pi r^2}$

$A = 8r^2 + 2\cancel{\pi}\cancel{r} \cdot \frac{1000}{\cancel{\pi} r^{\cancel{2}}} = 8r^2 + \frac{2000}{r} = 8r^2 + 2000r^{-1}$

$A' = 16r - 2000r^{-2} = 16r - \frac{2000}{r^2} = 0$ when $16r = \frac{2000}{r^2}$

$16r^3 = 2000 \rightarrow r^3 = 125 \rightarrow r = 5$ cm

$h = \frac{1000}{\pi r^2} = \frac{1000}{\pi \cdot 5^2} = \frac{1000}{25\pi} = \frac{40}{\pi}$ cm

$h : r = \frac{40}{\pi} : 5 = \boxed{\frac{8}{\pi} : 1}$

18. $V = lwh$

a) $l = \frac{15 - 2x}{2} = \frac{15}{2} - x$

$w = 10 - 2x$

$h = x$

$V = (\frac{15}{2} - x)(10 - 2x)x = (75 - 15x - 10x + 2x^2)x$

$\boxed{V = 75x - 25x^2 + 2x^3}$

b) $\frac{15}{2} - x > 0$ so $x < 7.5$

$10 - 2x > 0 \rightarrow 2x < 10 \rightarrow x < 5$ ← more restrictive than $x < 7.5$

$\boxed{0 < x < 5}$

c) Max on graph at $\boxed{x = 1.962 \text{ in}, V = 66.019 \text{ in}^3}$

d) $V' = 75 - 50x + 6x^2$

$6x^2 - 50x + 75 = 0$

$x = \frac{50 \pm \sqrt{50^2 - 4 \cdot 6 \cdot 75}}{2(6)} = \frac{50 \pm \sqrt{700}}{12}$ ↗ $6.371 \rightarrow$ not in $0 < x < 5$ ↘ $\boxed{1.962 \text{ in}}$

$V(1.962) = \boxed{66.019 \text{ in}^3}$

19. $V = \ell w h$

a) $\ell = \frac{36-4x}{2} = 18-2x$

$w = 24-2x$

$h = 2x$

$V = (18-2x)(24-2x)2x = (432-36x-48x+4x^2)2x$

$\boxed{V = 864x - 168x^2 + 8x^3}$

b) $18-2x > 0 \rightarrow 2x < 18 \rightarrow x < 9 \rightarrow$ more restrictive than $x < 12$

$24-2x > 0 \rightarrow 2x < 24 \rightarrow x < 12$

$\boxed{0 < x < 9}$

c) $V' = 864 - 336x + 24x^2$

$24x^2 - 336x + 864 = 0$

$$x = \frac{336 \pm \sqrt{336^2 - 4(24)(864)}}{2(24)} = \frac{336 \pm \sqrt{29{,}952}}{48} \begin{array}{l} \nearrow 10.606 \rightarrow \text{not in } 0 < x < 9 \\ \searrow \boxed{3.394 \text{ in}} \end{array}$$

$V(3.394) = 1{,}309.955 \text{ in}^3$

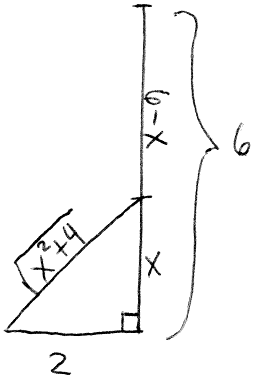
d) Max on graph at $(3.394, 1{,}309.955)$

e) $864x - 168x^2 + 8x^3 = 1120$

Find intersection on calc: $\boxed{x = 2 \text{ in}, x = 5 \text{ in}}$

f) Since the length is $18-2x$, x cannot exceed 9 bc length can't be negative. Since the width is $24-2x$, x cannot exceed 12 bc width can't be negative. x must also be greater than 0, otherwise there would be no cut out.

20.



$x^2+2^2=c^2$

$x^2+4=c^2$

$c=\sqrt{x^2+4}$

D = RT,

so $T=\frac{D}{R}$

$T(x)=\frac{\sqrt{x^2+4}}{2}+\frac{6-x}{5}=\frac{1}{2}(x^2+4)^{1/2}+\frac{6}{5}-\frac{1}{5}x$

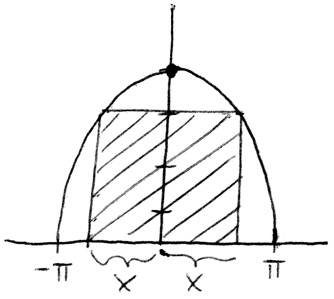
$T'(x)=\frac{1}{4}(x^2+4)^{-1/2}\cdot 2x-\frac{1}{5}=\frac{x}{2\sqrt{x^2+4}}-\frac{1}{5}$

$\frac{x}{2\sqrt{x^2+4}}-\frac{1}{5}=0 \rightarrow \frac{x}{2\sqrt{x^2+4}}=\frac{1}{5}$

$(5x)^2=(2\sqrt{x^2+4})^2 \rightarrow 25x^2=4(x^2+4)$

$25x^2=4x^2+16 \rightarrow 21x^2=16 \rightarrow x^2=16/21 \rightarrow x=\sqrt{16/21} \approx \boxed{0.873 \text{ mi}}$

21. $y=4\cos(0.5x)$ on $[-\pi,\pi]$



$A=bh=2x\cdot 4\cos(0.5x)$

$A=8x\cdot\cos(0.5x)$

$A'=8x\cdot -\sin(0.5x)\cdot 0.5+\cos(0.5x)\cdot 8$

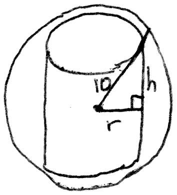
$A'=-4x\sin(0.5x)+8\cos(0.5x)=0$

at $x=1.7206672$ (find on graph)

$b=2x,\ h=4\cos(0.5x)$

$b=2\cdot x=\boxed{3.441},\ h=4\cos(0.5x)=\boxed{2.609},\ A=\boxed{8.978}$

22.



$r^2+h^2=10^2$

$h^2=100-r^2$

$h=\sqrt{100-r^2}$

Total height of cylinder (above & below center): $2h=2\sqrt{100-r^2}$

$V=\pi r^2h=\pi r^2\cdot 2\sqrt{100-r^2}=2\pi r^2(100-r^2)^{1/2}$

$V'=\cancel{2}\pi r^2\cdot\frac{1}{\cancel{2}}(100-r^2)^{-1/2}\cdot -2r+(100-r^2)^{1/2}\cdot 4\pi r$

$V'=\frac{-2\pi r^3}{\sqrt{100-r^2}}+\frac{4\pi r\sqrt{100-r^2}}{1}\cdot\frac{\sqrt{100-r^2}}{\sqrt{100-r^2}}=\frac{-2\pi r^3+4\pi r(100-r^2)}{\sqrt{100-r^2}}$

$V'=\frac{-2\pi r^3+400\pi r-4\pi r^3}{\sqrt{100-r^2}}=\frac{400\pi r-6\pi r^3}{\sqrt{100-r^2}}=0$ when $400\pi r-6\pi r^3=0$

$400\cancel{\pi}r=6\cancel{\pi}r^3 \rightarrow 400=6r^2 \rightarrow r^2=\frac{400}{6} \rightarrow r=\sqrt{\frac{400}{6}}=\frac{20}{\sqrt{6}}\approx\boxed{8.165 \text{ cm}}$

$h=2\sqrt{100-8.165^2}=\boxed{11.547 \text{ cm}},\ V=\pi r^2h=\boxed{2,418.399 \text{ cm}^3}$

30. $V = x^2 l$

$l + 4x = 108 \rightarrow l = 108 - 4x$

$V = x^2(108-4x) = 108x^2 - 4x^3$

$V' = 216x - 12x^2 = 0 \rightarrow 12x(18-x) = 0$

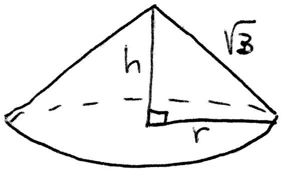
$12x = 0 \quad 18 - x = 0$

$x = 0 \quad \boxed{x = 18}$

$\boxed{\begin{matrix} W = 18\text{ in} \\ h = 18\text{ in} \end{matrix}}$

$l = 108 - 4(18) \rightarrow \boxed{l = 36\text{ in}}$

32.



$h^2 + r^2 = \sqrt{3}^2 \rightarrow h^2 + r^2 = 3 \rightarrow r^2 = 3 - h^2$

$V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi(3-h^2)h = \frac{1}{3}\pi(3h - h^3)$

$V' = \frac{1}{3}\pi(3 - 3h^2) = 0 \rightarrow 3 - 3h^2 = 0 \rightarrow 3h^2 = 3 \rightarrow h^2 = 1$

$\boxed{h = 1\text{ m}}$

$r^2 = 3 - h^2 \rightarrow r = \sqrt{3-h^2} = \sqrt{3-1^2} = \sqrt{3-1} = \boxed{\sqrt{2}\text{ m}}$

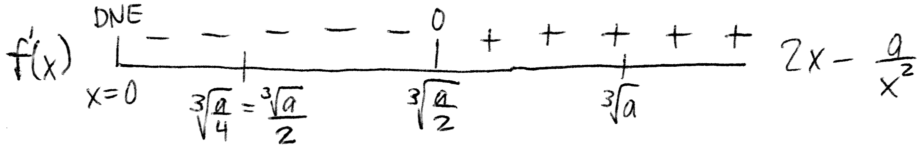
$V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi \cdot \cancel{\sqrt{2}}^{\,2} \cdot 1 = \frac{1}{3} \cdot \pi \cdot 2 = \boxed{\frac{2\pi}{3}\text{ m}^3}$

34. $f(x) = x^2 + \frac{a}{x} = x^2 + ax^{-1}$

$f'(x) = 2x - ax^{-2} = 2x - \frac{a}{x^2}$

$f'(x)$ DNE when $x = 0$

$2x - \frac{a}{x^2} = 0 \rightarrow 2x = \frac{a}{x^2} \rightarrow 2x^3 = a \rightarrow x^3 = \frac{a}{2} \rightarrow x = \sqrt[3]{\frac{a}{2}}$



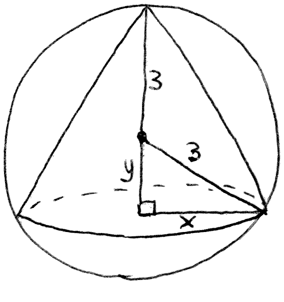
$f'(\sqrt[3]{a}) = 2\sqrt[3]{a} - \frac{a}{(\sqrt[3]{a})^2} = 2\sqrt[3]{a} - \frac{a}{a^{2/3}} = 2\sqrt[3]{a} - a^{1/3} = 2\sqrt[3]{a} - \sqrt[3]{a} = \sqrt[3]{a} = +$

$f'\left(\frac{\sqrt[3]{a}}{2}\right) = \cancel{2}\frac{\sqrt[3]{a}}{\cancel{2}} - \frac{a}{(\sqrt[3]{a/2})^2} = \sqrt[3]{a} - \frac{a}{\frac{a^{2/3}}{4}} = \sqrt[3]{a} - \frac{4a}{a^{2/3}} = \sqrt[3]{a} - 4a^{1/3} = \sqrt[3]{a} - 4\sqrt[3]{a} = -3\sqrt[3]{a}$

$= -$

Sign of f' only changes at $x = \sqrt[3]{\frac{a}{2}}$. The sign of f' changes from negative to positive there, so it must be a minimum.

36.



$x^2+y^2=3^2 \rightarrow x^2+y^2=9 \rightarrow x=9-y$

$V=\frac{1}{3}\pi x^2(3+y)=\frac{1}{3}\pi(9-y^2)(3+y)$

$V=\frac{1}{3}\pi(27+9y-3y^2-y^3)=9\pi+3\pi y-\pi y^2-\frac{1}{3}\pi y^3$

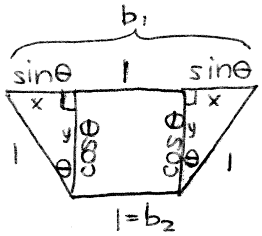
$V'=3\pi-2\pi y-\pi y^2=\pi(3-2y-y^2)=0 \rightarrow 3-2y-y^2=0$

$y=\frac{2\pm\sqrt{2^2-4(-1)(3)}}{2(-1)}=\frac{2\pm\sqrt{16}}{-2}=\frac{2\pm4}{-2}$ $\nearrow \frac{6}{-2}=-3 \rightarrow$ can't be negative; $\searrow \frac{-2}{-2}=1 \rightarrow \boxed{y=1}$

$x^2=9-y^2 \rightarrow x=\sqrt{9-y^2} \rightarrow x=\sqrt{9-1^2} \rightarrow \boxed{x=\sqrt{8}}$

$V=\frac{1}{3}\pi\cdot\sqrt{8}^2\cdot(3+1)=\frac{1}{3}\pi\cdot8\cdot4=\boxed{\frac{32\pi}{3}}$

47.



$\sin\theta=\frac{x}{1} \rightarrow x=\sin\theta$

$\cos\theta=\frac{y}{1} \rightarrow y=\cos\theta$

Area of trapezoid $=\frac{1}{2}h(b_1+b_2)=\frac{1}{2}\cos\theta(1+\sin\theta+\sin\theta+1)$

Area $=\frac{1}{2}\cos\theta(2+2\sin\theta)=\cos\theta(1+\sin\theta)$

Volume = Area x Length $=\cos\theta(1+\sin\theta)\cdot20=20\cos\theta(1+\sin\theta)$

$V'=20\cos\theta\cdot\cos\theta+(1+\sin\theta)(-20\sin\theta)$

$V'=20\cos^2\theta-20\sin\theta-20\sin^2\theta=0$ at $\theta=0.52359878=\boxed{\frac{\pi}{6}}$