Section 5.5 : 1-7 odd, 8-11 all, 13-33 e.o.o., 41, 53, 55, 66, 71

1. a) $f(x) = x^3 - 2x + 3,\ a = 2$

$f(2) = 8 - 4 + 3 = 7 \rightarrow (2,7)$

$f'(x) = 3x^2 - 2$

$f'(2) = 12 - 2 = 10$

$y - 7 = 10(x-2)$

$L(x) = \boxed{10x - 13}$

b) $f(2.1) = 8.061$

$L(2.1) = 8$

Difference $= 0.061$

$0.061 < 0.1$

Differs by $\boxed{< 10^{-1}}$

3. a) $f(x) = x + \frac{1}{x},\ a = 1$

$f(1) = 1 + 1 = 2 \rightarrow (1,2)$

$f'(x) = 1 - \frac{1}{x^2}$

$f'(1) = 1 - 1 = 0$

$y - 2 = 0(x-1)$

$L(x) = \boxed{2}$

b) $f(1.1) = 2.00909$

$L(1.1) = 2$

Difference $= 0.00909$

$0.00909 < 0.01$

Differs by $\boxed{< 10^{-2}}$

5. a) $f(x) = \tan x,\ a = \pi$

$f(\pi) = \tan\pi = \frac{0}{-1} = 0 \rightarrow (\pi, 0)$

$f'(x) = \sec^2 x$

$f'(\pi) = (\sec\pi)^2 = (-1)^2 = 1$

$y - 0 = 1(x - \pi)$

$L(x) = \boxed{x - \pi}$

b) $f(\pi + 0.1) = 0.100335$

$L(\pi + 0.1) = 0.1$

Difference $= 0.000335$

$0.000335 < 0.001$

Differs by $\boxed{< 10^{-3}}$

7. $f(x) = (1+x)^k$ at $x = 0$

$f(0) = (1+0)^k = 1^k = 1 \rightarrow (0,1)$

$f'(x) = k(1+x)^{k-1}$

$f'(0) = k(1+0)^{k-1} = k \cdot 1 = k$

$y - 1 = k(x - 0)$

$L(x) = \boxed{kx + 1}$

8. a) $(1.002)^{100} = (1 + \underbrace{0.002}_{x})^{\overset{100}{k}} \approx 1 + kx = 1 + 100(0.002) = \boxed{1.2}$

Exact $= 1.22116 \rightarrow$ Difference $= 0.02116 \rightarrow$ Differs by $\boxed{< 10^{-1}}$

8\. b) $1.009^{1/3} = (\underbrace{1+0.009}_{x})^{\overset{1/3}{k}} \approx 1+kx = 1+\frac{1}{3}(0.009) = \boxed{1.003}$

Exact $= 1.002991 \rightarrow$ Difference $= 8.955\times10^{-6} \rightarrow$ Differs by $\boxed{< 10^{-5}}$

9\. a) $f(x) = (1-x)^6 = (1+\underbrace{(-x)}_{x})^{\overset{6}{k}} \approx 1+kx = 1+6(-x) = \boxed{1-6x}$

b) $f(x) = \frac{2}{1-x} = 2(1\underbrace{-x}_{x})^{\overset{-1}{k}} \approx 2(1+kx) = 2(1+(-1)(-x)) = 2(1+x) = \boxed{2+2x}$

c) $f(x) = (1+\underbrace{x}_{x})^{\overset{-1/2}{k}} \approx 1+kx = 1+\frac{-1}{2}x = \boxed{1-\frac{1}{2}x}$

10\. a) $f(x) = (4+3x)^{1/3} = (4(1+3/4x))^{1/3} = 4^{1/3}(1+\underbrace{3/4x}_{x})^{\overset{1/3}{k}}$

$4^{1/3}(1+kx) = 4^{1/3}(1+\frac{1}{\cancel{3}}(\frac{\cancel{3}}{4}x)) = \boxed{4^{1/3}(1+\frac{1}{4}x)}$

b) $f(x) = (2+x^2)^{1/2} = (2(1+\frac{1}{2}x^2))^{1/2} = 2^{1/2}(1+\underbrace{\tfrac{1}{2}x^2}_{x})^{\overset{1/2}{k}}$

$\sqrt{2}(1+kx) = \sqrt{2}(1+\frac{1}{2}\cdot\frac{1}{2}x^2) = \boxed{\sqrt{2}(1+\frac{1}{4}x^2)}$

c) $f(x) = (1\underbrace{-\tfrac{1}{2+x}}_{x})^{\overset{2/3}{k}} \approx 1+kx = 1+\frac{2}{3}\cdot\frac{-1}{2+x} = \boxed{1-\frac{2}{6+3x}}$

11\. $\sqrt{101}$

$\sqrt{100} = 10 \rightarrow y = \sqrt{x} = x^{1/2}, \ (100, 10)$

$y' = \frac{1}{2}x^{-1/2} = \frac{1}{2\sqrt{x}}$

$y'(100) = \frac{1}{2\sqrt{100}} = \frac{1}{2\cdot10} = \frac{1}{20}$

$y-10 = \frac{1}{20}(x-100) \rightarrow y-10 = \frac{1}{20}x-5 \rightarrow L(x) = \frac{1}{20}x+5$

$L(101) = \frac{1}{20}(101)+5 = \boxed{10.05}$

13\. $\sqrt[3]{998}$

$\sqrt[3]{1000} = 10 \rightarrow y = \sqrt[3]{x} = x^{1/3}, \ (1000, 10)$

$y' = \frac{1}{3}x^{-2/3} = \frac{1}{3x^{2/3}}$

$y'(1000) = \frac{1}{3\cdot1000^{2/3}} = \frac{1}{3\cdot100} = \frac{1}{300}$

$y-10 = \frac{1}{300}(x-1000) \rightarrow y-10 = \frac{1}{300}x-\frac{10}{3} \rightarrow L(x) = \frac{1}{300}x+\frac{20}{3}$

$L(998) = \frac{1}{300}(998)+\frac{20}{3} = \boxed{9.993}$

17. $y = x^2 \ln x,\ x=1,\ dx = 0.01$

a) $dy = (x^2 \cdot \frac{1}{x} + \ln x \cdot 2x)\,dx = \boxed{(x + 2x \cdot \ln x)dx}$

b) $dy = (1 + 2 \cdot \ln 1)\,0.01 = (1+0)\,0.01 = \boxed{0.01}$

21. $y + xy - x = 0,\ x = 0,\ dx = 0.01$

a) $y + xy = x \rightarrow y(1+x) = x \rightarrow y = \frac{x}{1+x}$

$$dy = \frac{(1+x)1 - x(1)}{(1+x)^2}\,dx = \frac{1 + \cancel{x} - \cancel{x}}{(1+x)^2}\,dx = \boxed{\frac{1}{(1+x)^2}\,dx}$$

b) $dy = \frac{1}{(1+0)^2}(0.01) = \frac{1}{1} \cdot 0.01 = \boxed{0.01}$

25. $d(\tan^{-1} 4x) = \frac{1}{1+(4x)^2} \cdot 4\,dx = \boxed{\frac{4}{1+16x^2}\,dx}$

29. $f(x) = \frac{1}{x},\ a = 0.5,\ dx = 0.05$

a) $f(0.55) = \frac{1}{0.55} = \frac{1}{11/20} = \frac{20}{11}$

$f(0.5) = \frac{1}{0.5} = \frac{1}{1/2} = 2 = \frac{22}{11}$

$\Delta f = \frac{20}{11} - \frac{22}{11} = \boxed{\frac{-2}{11}}$

b) $f'(x) = \frac{-1}{x^2} \rightarrow f'(0.5) = \frac{-1}{(1/2)^2} = \frac{-1}{1/4} = -4$

$df = f'(0.5) \cdot dx = -4(0.05) = -4 \cdot \frac{1}{20} = \boxed{\frac{-1}{5}}$

c) $|\Delta f - df| = \left|\frac{-2}{11} - \frac{-1}{5}\right| = \left|\frac{-10}{55} + \frac{11}{55}\right| = \boxed{\frac{1}{55}}$

33. $V = x^3,\ x = 10,\ dx = 0.05$

$\boxed{dV = 3x^2 dx}$

$dV = 3(10)^2\, 0.05 = 300(0.05) = \boxed{15\ cm^3}$

41. $f(0) = 1,\ f'(x) = \cos(x^2)$

a) $f'(0) = \cos 0 = 1$

$y - 1 = 1(x - 0) \rightarrow L(x) = \boxed{x+1}$

b) $L(0.1) = 0.1 + 1 = \boxed{1.1}$

c) $f'(x)$ is largest when $x=0$. Therefore, $f'(x)$ is decreasing on $[0, 0.1]$. Therefore, the actual value at $x = 0.1$ is $\boxed{\text{less}}$ than 1.1.

53. $x^3+x-1=0$

$x \approx 0.8$

$$x_{n+1} = x_n - \frac{x_n^3+x_n-1}{3x_n^2+1}$$

$x_1 = 0.8$
$x_2 = 0.693151$
$x_3 = 0.682427$
$x_4 = \boxed{0.682328}$
$x_5 = \boxed{0.682328}$

55. $x^2-2x+1-\sin x = 0$

$x \approx 0.4$ and $x \approx 2$

$$x_{n+1} = x_n - \frac{x_n^2-2x_n+1-\sin x_n}{2x_n-2-\cos x_n}$$

$x_1 = 0.4$
$x_2 = 0.386130$
$x_3 = \boxed{0.386237}$
$x_4 = \boxed{0.386237}$

$x_1 = 2$
$x_2 = 1.962460$
$x_3 = 1.961570$
$x_4 = \boxed{1.961569}$
$x_5 = \boxed{1.961569}$

66. $Q(x) = b_0 + b_1(x-a) + b_2(x-a)^2$

$Q(a) = f(a)$, $Q'(a) = f'(a)$, $Q''(a) = f''(a)$

a) $Q(a)$: $b_0 + b_1(a-a) + b_2(a-a)^2 = f(a) \to b_0 + 0 + 0 = f(a) \to \boxed{b_0 = f(a)}$

$Q'(x) = b_1 + 2b_2(x-a)$

$Q'(a)$: $b_1 + 2b_2(a-a) = f'(a) \to b_1 + 0 = f'(a) \to \boxed{b_1 = f'(a)}$

$Q''(x) = 2b_2$

$Q''(a)$: $2b_2 = f''(a) \to \boxed{b_2 = \frac{1}{2}f''(a)}$

b) $f(x) = \frac{1}{1-x}$ at $x=0$ $\quad f(x) = (1-x)^{-1}$

$f(0) = \frac{1}{1-0} = 1$

$f'(x) = -1(1-x)^{-2}(-1) = \frac{1}{(1-x)^2} \to f'(0) = \frac{1}{(1-0)^2} = 1$

$f''(x) = -2(1-x)^{-3}(-1) = \frac{2}{(1-x)^3} \to f''(0) = \frac{2}{(1-0)^3} = 2$

$Q(0) = f(0) + f'(0)(x-\cancel{0}) + \frac{1}{2}f''(0)(x-\cancel{0})^2 = \boxed{1+x+x^2}$

c) We cannot distinguish a difference. The graphs appear the same around $(0,1)$.

d) $g(x) = \frac{1}{x}$ at $x=1$

$g(1) = 1$

$g'(x) = \frac{-1}{x^2} \to g'(1) = \frac{-1}{1^2} = -1$

$g''(x) = \frac{2}{x^3} \to g''(1) = \frac{2}{1^3} = 2$

$Q(1) = \boxed{1-(x-1)+(x-1)^2}$

The graphs appear the same around $(1,1)$.

66. e) $h(x) = (1+x)^{1/2}$, $x = 0$

$h(0) = (1+0)^{1/2} = 1$

$h'(x) = \frac{1}{2}(1+x)^{-1/2} \rightarrow h'(0) = \frac{1}{2} \cdot 1^{-1/2} = \frac{1}{2}$

$h''(x) = -\frac{1}{4}(1+x)^{-3/2} \rightarrow h''(0) = -\frac{1}{4}(1+0)^{-3/2} = \frac{-1}{4}$

$Q(0) = 1 + \frac{1}{2}(x - \cancel{0}) + \frac{1}{2} \cdot \frac{-1}{4}(x - \cancel{0})^2 = \boxed{1 + \frac{1}{2}x - \frac{1}{8}x^2}$

The graphs appear the same around $(0,1)$.

f) $f(0) = 1$, $f'(0) = 1 \rightarrow y - 1 = 1(x-0) \rightarrow L(x) = \boxed{x+1}$

$g(1) = 1$, $g'(1) = -1 \rightarrow y - 1 = -1(x-1) \rightarrow L(x) = \boxed{-x+2}$

$h(0) = 1$, $h'(0) = \frac{1}{2} \rightarrow y - 1 = \frac{1}{2}(x-0) \rightarrow L(x) = \boxed{\frac{1}{2}x + 1}$

71. $f(x) = \sqrt{x+1} + \sin x$ at $x = 0$

$f(0) = \sqrt{1} + \sin 0 = 1 + 0 = 1 \rightarrow (0,1)$

$f'(x) = \frac{1}{2}(x+1)^{-1/2} + \cos x$

$f'(0) = \frac{1}{2} \cdot 1^{-1/2} + \cos 0 = \frac{1}{2} + 1 = \frac{3}{2}$

$y - 1 = \frac{3}{2}(x-0)$

$L(x) = \boxed{\frac{3}{2}x + 1}$

$g(x) = \sqrt{x+1}$ at $x = 0$

$g(0) = \sqrt{0+1} = \sqrt{1} = 1 \rightarrow (0,1)$

$g'(x) = \frac{1}{2}(x+1)^{-1/2} \rightarrow g'(0) = \frac{1}{2} \cdot 1^{-1/2} = \frac{1}{2}$

$y - 1 = \frac{1}{2}(x-0)$

$L(x) = \frac{1}{2}x + 1$

$h(x) = \sin x$ at $x = 0$

$h(0) = \sin 0 = 0 \rightarrow (0,0)$

$h'(x) = \cos x \rightarrow h'(0) = \cos 0 = 1$

$y - 0 = 1(x-0)$

$L(x) = x$

$\frac{1}{2}x + 1 + x = \boxed{\frac{3}{2}x + 1}$

The sum of the individual linearizations is equivalent to the original linearization of the sum of the functions.