# Section 5.6 - Part 2: 23-35 all, 42-44 all

23. $y = \dfrac{10}{1+x^2} = 10(1+x^2)^{-1}$, $\dfrac{dx}{dt} = 3$ cm/s

a) $\dfrac{dy}{dt} = -10(1+x^2)^{-2} \cdot 2x \dfrac{dx}{dt} = \dfrac{-20x \frac{dx}{dt}}{(1+x^2)^2}$

Ax $x=-2$: $\dfrac{dy}{dt} = \dfrac{-20(-2)(3)}{(1+(-2)^2)^2} = \dfrac{120}{25} = \boxed{\dfrac{24}{5} \text{ cm/s}}$

b) Ax $x=0$: $\dfrac{dy}{dt} = \dfrac{-20(0)(3)}{(1+0^2)^2} = \dfrac{0}{1} = \boxed{0 \text{ cm/s}}$

c) At $x=20$: $\dfrac{dy}{dt} = \dfrac{-20(20)(3)}{(1+20^2)^2} = \dfrac{-12{,}000}{401^2} = \boxed{-0.00746 \text{ cm/s}}$

24. $y = x^3 - 4x$, $\dfrac{dx}{dt} = -2$ cm/s

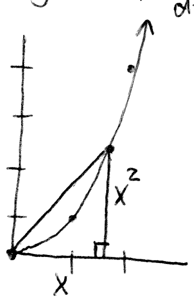
a) $\dfrac{dy}{dt} = 3x^2\dfrac{dx}{dt} - 4\dfrac{dx}{dt}$

At $x=-3$: $\dfrac{dy}{dt} = 3(-3)^2(-2) - 4(-2) = -54+8 = \boxed{-46 \text{ cm/s}}$

b) At $x=1$: $\dfrac{dy}{dt} = 3(1)^2(-2) - 4(-2) = -6+8 = \boxed{2 \text{ cm/s}}$

c) At $x=4$: $\dfrac{dy}{dt} = 3(4)^2(-2) - 4(-2) = -96+8 = \boxed{-88 \text{ cm/s}}$

25. $y = x^2$, $\dfrac{dx}{dt} = 10$ m/s, $x=3$, $\dfrac{d\theta}{dt} = ?$



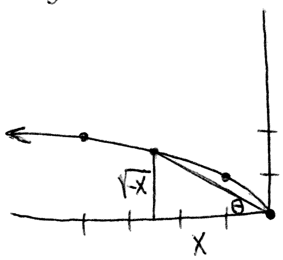

$\tan\theta = \dfrac{x^2}{x}$

$\tan\theta = x$

$\theta = \tan^{-1} x$

$\dfrac{d\theta}{dt} = \dfrac{1}{1+x^2} \cdot \dfrac{dx}{dt} = \dfrac{1}{1+3^2} \cdot 10 = \dfrac{10}{10} = \boxed{1 \text{ radian/s}}$

26. $y = \sqrt{-x}$ can only have $x \le 0$




$\frac{dx}{dt} = -8$ m/s, $x = -4$ m, $\frac{d\theta}{dt} = ?$

$\tan\theta = \frac{\sqrt{-x}}{x} \rightarrow \tan\theta = \frac{(-x)^{1/2}}{-(-x)} \rightarrow \tan\theta = \frac{-1}{\sqrt{-x}} \rightarrow \tan\theta = -(-x)^{-1/2}$

$\theta = \tan^{-1}\left(-(-x)^{-1/2}\right)$

$$\frac{d\theta}{dt} = \frac{1}{1+\left(-\frac{1}{\sqrt{-x}}\right)^2} \cdot \frac{1}{2}(-x)^{-3/2} \cdot -\frac{dx}{dt} = \frac{1}{1+\frac{1}{-x}} \cdot \frac{-1/2\, dx/dt}{(-x)^{3/2}}$$

$$\frac{d\theta}{dt} = \frac{1}{1+\frac{1}{-(-4)}} \cdot \frac{-1/2(-8)}{(--4)^{3/2}} = \frac{1}{1+1/4} \cdot \frac{4}{8} = \frac{1}{5/4} \cdot \frac{1}{2} = \frac{4}{5} \cdot \frac{1}{2} = \frac{4}{10} = \boxed{\frac{2}{5} \text{ radian/s}}$$

27. $\frac{dV}{dt} = -8$ mL/min $= -8$ cm$^3$/min

$\frac{dA}{dt} = ?$ when $d = 20$ cm $\rightarrow$ $r = 10$ cm

$V = \frac{4}{3}\pi r^3$

$$\frac{dV}{dt} = 4\pi r^2 \cdot \frac{dr}{dt} \rightarrow \frac{dr}{dt} = \frac{dV/dt}{4\pi r^2} = \frac{-8}{4\pi \cdot 10^2} = \frac{-8}{400\pi} = \frac{-1}{50\pi} \text{ cm/min}$$

$A = 4\pi r^2$

$$\frac{dA}{dt} = 8\pi r \frac{dr}{dt} = 8\cancel{\pi} \cdot 10 \cdot \frac{-1}{50\cancel{\pi}} = \frac{-80}{50} = \frac{-8}{5} = \boxed{-1.6 \text{ cm}^2/\text{min}}$$ (surface area decr.)

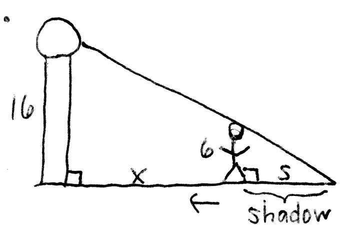
28. $\frac{dx}{dt} = -1$ m/s, $\frac{dy}{dt} = -5$ m/s, $x = 5$, $y = 12$

$D = \sqrt{x^2+y^2} = (x^2+y^2)^{1/2}$

$$\frac{dD}{dt} = \frac{1}{2}(x^2+y^2)^{-1/2}\left(2x\frac{dx}{dt} + 2y\frac{dy}{dt}\right) = \frac{x\frac{dx}{dt} + y\frac{dy}{dt}}{\sqrt{x^2+y^2}} = \frac{5(-1)+12(-5)}{\sqrt{12^2+5^2}} = \frac{-5-60}{\sqrt{169}} = \frac{-65}{13}$$

$\frac{dD}{dt} = \boxed{-5 \text{ m/s}}$

29.



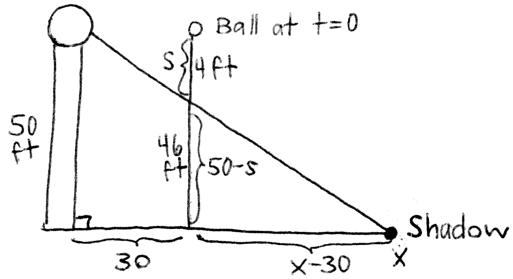

$\frac{dx}{dt} = -5$ ft/s

$\frac{ds}{dt} = ?$ when $x = 10$ ft

$$\frac{16}{x+s} = \frac{6}{s} \rightarrow 16s = 6x + 6s \rightarrow 10s = 6x \rightarrow s = \frac{6}{10}x \rightarrow s = \frac{3}{5}x$$

$$\frac{ds}{dt} = \frac{3}{5}\frac{dx}{dx} = \frac{3}{\cancel{5}}(-\cancel{5}) = \boxed{-3 \text{ ft/s}}$$

30.




$s = 16t^2$

$s(\tfrac{1}{2}) = 16(\tfrac{1}{2})^2 = 16 \cdot 1/4 = 4$ ft down in $\tfrac{1}{2}$ sec

$\frac{dx}{dt} = ?$

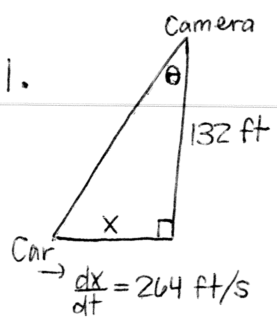
$\frac{ds}{dt} = 32t$

$$\frac{50}{x} = \frac{50-s}{x-30} \rightarrow \cancel{50x} - 1500 = \cancel{50x} - sx$$

$$sx = 1500 \rightarrow x = \frac{1500}{s} = 1500s^{-1}$$

$$\frac{dx}{dt} = -1500s^{-2}\frac{ds}{dt} = \frac{-1500}{s^2} \cdot \frac{ds}{dt} = \frac{-1500}{4^2} \cdot 32(\tfrac{1}{2}) = \frac{-1500}{\cancel{16}} \cdot \cancel{16} = \boxed{-1500 \text{ ft/s}}$$

31.




$$\tan\theta = \frac{x}{132} \rightarrow \theta = \tan^{-1}(\tfrac{1}{132}x)$$

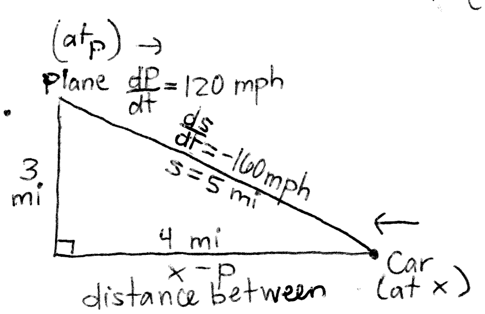
$$\frac{d\theta}{dt} = \frac{1}{1+(\frac{x}{132})^2} \cdot \frac{1}{132} \cdot \frac{dx}{dt}$$

At $x=0$: $\frac{d\theta}{dt} = \frac{1}{1+\cancel{0}} \cdot \frac{1}{132} \cdot 264 = \frac{264}{132} = \boxed{2 \text{ radians/s}}$

$\tfrac{1}{2}$ second later: $x = \frac{264 \text{ ft}}{\cancel{\text{sec}}} \cdot \frac{\tfrac{1}{2}\ \cancel{\text{sec}}}{1} = 132$ ft

$$\frac{d\theta}{dt} = \frac{1}{1+(\frac{132}{132})^2} \cdot \frac{1}{132} \cdot 264 = \frac{1}{2} \cdot \frac{264}{132} = \frac{1}{2} \cdot 2 = \boxed{1 \text{ radian/s}}$$

32.



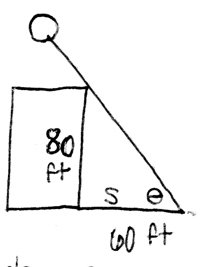

$$(x-p)^2 + 3^2 = s^2$$

$$\cancel{2}(x-p)(\tfrac{dx}{dt} - \tfrac{dP}{dt}) = \cancel{2}s\tfrac{ds}{dt}$$

$$4(\tfrac{dx}{dt} - 120) = 5(-160)$$

$$\frac{dx}{dt} - 120 = -200 \rightarrow \frac{dx}{dt} = \boxed{-80 \text{ mph}} \text{ (to the left)}$$

33.




$\frac{ds}{dt} = ?$

$\frac{d\theta}{dt} = 0.27°/\text{min} \cdot \frac{\pi}{180}$

$\frac{d\theta}{dt} = 0.00471$ rad/min

$\tan\theta = \frac{80}{60} \rightarrow \tan\theta = \frac{4}{3} \rightarrow \theta = \tan^{-1}(4/3)$ when $s = 60$ ft

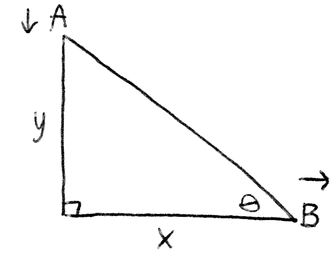
$$\tan\theta = \frac{80}{s} \rightarrow \tan\theta = 80s^{-1}$$

$$\sec^2\theta \frac{d\theta}{dt} = -80s^{-2}\frac{ds}{dt}$$

$$\sec^2\theta \frac{d\theta}{dt} = \frac{-80}{s^2} \cdot \frac{ds}{dt}$$

$$\frac{ds}{dt} = \frac{\sec^2\theta \cdot d\theta/dt \cdot s^2}{-80} = \frac{(\sec(\tan^{-1}4/3))^2 \cdot 0.00471 \cdot 60^2}{-80}$$

$$\frac{ds}{dt} = -0.589 \text{ ft/min} = \boxed{-7.069 \text{ in/min}} \text{ (length decreasing)}$$

34. 

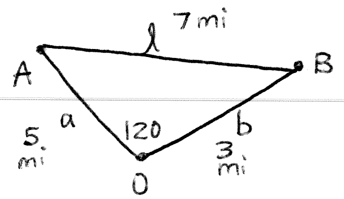
$\frac{dy}{dt} = -2\ m/s,\ \frac{dx}{dt} = 1\ m/s,\ y = 10\ m,\ x = 20\ m$

$\tan\theta = \frac{10}{20} \to \tan\theta = \frac{1}{2} \to \theta = \tan^{-1}(1/2)$

$\tan\theta = \frac{y}{x}$

$$\sec^2\theta \frac{d\theta}{dt} = \frac{x\,dy/dt - y\,dx/dt}{x^2} \to \frac{d\theta}{dt} = \frac{x\,dy/dt - y\,dx/dt}{x^2 \cdot \sec^2\theta}$$

$$\frac{d\theta}{dt} = \frac{20(-2) - 10(1)}{20^2 \cdot \sec^2(\tan^{-1} 1/2)} = -0.1 \text{ radian/s} \times \frac{360°}{2\pi} = -5.730 \text{ deg/s} \approx \boxed{-6 \text{ deg/s}}$$

35. 

$\frac{dA}{dt} = 14$ knots, $\frac{dB}{dt} = 21$ knots

$l^2 = a^2 + b^2 - 2ab\cos O$

$l^2 = 5^2 + 3^2 - 2 \cdot 5 \cdot 3 \cdot \cos 120 = 25 + 9 - 30(-1/2) = 25 + 9 + 15$

$l^2 = 49 \to l = 7$

$l^2 = a^2 + b^2 - 2ab(-1/2) \to l^2 = a^2 + b^2 + ab$

$$2l\frac{dl}{dt} = 2a\frac{dA}{dt} + 2b\frac{dB}{dt} + a\frac{dB}{dt} + b\frac{dA}{dt}$$

$$\frac{dl}{dt} = \frac{2a\frac{dA}{dt} + 2b\frac{dB}{dt} + a\frac{dB}{dt} + b\frac{dA}{dt}}{2l} = \frac{2(5)(14) + 2(3)(21) + (5)(21) + (3)(14)}{2(7)}$$

$$\frac{dl}{dt} = \frac{140 + 126 + 105 + 42}{14} = \frac{413}{14} = \boxed{29.5 \text{ knots}}$$

42. $\frac{dV}{dt} = 10\ in^3/min$ out of cone, into cylinder

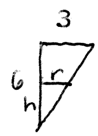
Cone: $h = 6$ in, $d = 6$ in, $r = 3$ in

Cylinder: $d = 6$ in, $r = 3$ in

a) $V = \pi r^2 h = \pi \cdot 3^2 \cdot h \to V = 9\pi h$

$$\frac{dV}{dt} = 9\pi\frac{dh}{dt} \to \frac{dh}{dt} = \frac{dV/dt}{9\pi} = \frac{10}{9\pi} = \boxed{0.354 \text{ in/min}} \text{ (level rising)}$$

b) $V = \frac{1}{3}\pi r^2 h$ ($r = \frac{1}{2}h$)




$\frac{3}{6} = \frac{r}{h} \to 3h = 6r \to r = \frac{1}{2}h$

$$V = \frac{1}{3}\pi\left(\frac{1}{2}h\right)^2 h = \frac{1}{3}\pi \cdot \frac{1}{4}h^2 h \to V = \frac{1}{12}\pi h^3$$

$$\frac{dV}{dt} = \frac{1}{4}\pi h^2\frac{dh}{dt} \to \frac{dh}{dt} = \frac{dV/dt}{\frac{1}{4}\pi h^2} = \frac{-10}{\left(\frac{1}{4}\pi \cdot 5^2\right)} = \boxed{-0.509 \text{ in/min}} \text{ (level falling)}$$

43. a) $r(x) = 9x$, $c(x) = x^3 - 6x^2 + 15x$, $x = 2$, $\frac{dx}{dt} = 0.1$

$$r = 9x \rightarrow \frac{dr}{dt} = 9\frac{dx}{dt} = 9(0.1) = \boxed{0.9}$$

$$c = x^3 - 6x^2 + 15x \rightarrow \frac{dc}{dt} = 3x^2\frac{dx}{dt} - 12x\frac{dx}{dt} + 15\frac{dx}{dt}$$

$$\frac{dc}{dt} = 3\cdot 2^2\cdot 0.1 - 12(2)\cdot 0.1 + 15(0.1) = 1.2 - 2.4 + 1.5 = \boxed{0.3}$$

$$p = r - c$$

$$\frac{dp}{dt} = \frac{dr}{dt} - \frac{dc}{dt} = 0.9 - 0.3 = \boxed{0.6}$$

b) $r(x) = 70x$, $c(x) = x^3 - 6x^2 + 45x^{-1}$, $x = 1.5$, $\frac{dx}{dt} = 0.05$

$$r = 70x \rightarrow \frac{dr}{dt} = 70\frac{dx}{dt} = 70(0.05) = \boxed{3.5}$$

$$c = x^3 - 6x^2 + 45x^{-1}$$

$$\frac{dc}{dt} = 3x^2\frac{dx}{dt} - 12x\frac{dx}{dt} - \frac{45}{x^2}\frac{dx}{dt} = 3\cdot 1.5^2\cdot 0.05 - 12\cdot 1.5\cdot 0.05 - \frac{45}{1.5^2}\cdot 0.05 = \boxed{-1.5625}$$

$$p = r - c$$

$$\frac{dp}{dt} = \frac{dr}{dt} - \frac{dc}{dt} = 3.5 - -1.5625 = \boxed{5.0625}$$

14. $y = \frac{Q}{D}$, $Q = 233$, $D = 41$, $\frac{dD}{dt} = -2$, $\frac{dQ}{dt} = 0$

$$\frac{dy}{dt} = \frac{D\frac{dQ}{dt} - Q\frac{dD}{dt}}{D^2} = \frac{41(0) - 233(-2)}{41^2} = \frac{466}{41^2} = 0.277 \text{ L/min per min}$$

$$\boxed{0.277 \text{ L/min}^2}$$