Section 6.3: 1–41 e.o.o.

1. $\int_1^2 f(x)dx = -4$, $\int_1^5 f(x)\,dx = 6$, $\int_1^5 g(x)\,dx = 8$

a) $\int_2^2 g(x)dx = \boxed{0}$ (x's haven't gone anywhere → width = 0)

b) $\int_5^1 g(x)dx = -\int_1^5 g(x)dx = \boxed{-8}$

c) $\int_1^2 3f(x)dx = 3\int_1^2 f(x)dx = 3(-4) = \boxed{-12}$

d) $\int_2^5 f(x)dx = \int_1^5 f(x)dx - \int_1^2 f(x)dx = 6 - -4 = \boxed{10}$

e) $\int_1^5 [f(x)-g(x)]dx = \int_1^5 f(x)dx - \int_1^5 g(x)dx = 6-8 = \boxed{-2}$

f) $\int_1^5 [4f(x)-g(x)]dx = 4\int_1^5 f(x)dx - \int_1^5 g(x)dx = 4(6)-8 = 24-8 = \boxed{16}$

5. $\int_0^3 f(z)dz = 3$ and $\int_0^4 f(z)dz = 7$

a) $\int_3^4 f(z)dz = \int_0^4 f(z)dz - \int_0^3 f(z)dz = 7-3 = \boxed{4}$

b) $\int_4^3 f(z)dz = -\int_3^4 f(z)dz = \boxed{-4}$

9. $f(x) \geq 0$ on $[a,b]$

Min Area $=$ Min y value $\times \Delta x = 0(b-a) = 0$

$\int_a^b f(x)dx \geq$ Min Area so $\boxed{\int_a^b f(x)dx \geq 0}$.

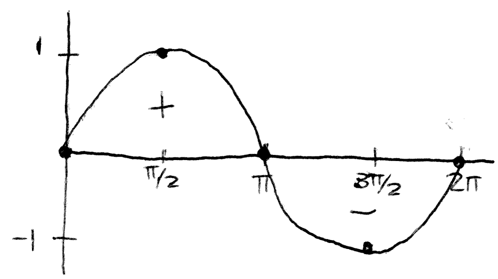
13. $y = -3x^2 - 1$ on $[0,1]$

$\int_0^1 (-3x^2-1)dx = \text{NINT}(-3x^2-1, x, 0, 1) \to$ area $= -2$

Avg. y value $= \frac{\text{Area}}{\Delta x} = \frac{-2}{1-0} = \frac{-2}{1} = \boxed{-2}$

$-3x^2 - 1 = -2 \to -3x^2 = -1 \to x^2 = 1/3 \to \boxed{x = \sqrt{1/3}}$

17. $f(t) = \sin t$, $[0, 2\pi]$



Avg. y value $= \frac{\text{Area}}{\Delta x} = \frac{0}{2\pi} = \boxed{0}$

21. $\int_0^1 e^x dx = e^x \Big|_0^1 = e^1 - e^0 = \boxed{e-1}$

25. $\int_{-2}^{6} 5dx = 5x \Big|_{-2}^{6} = 5(6) - 5(-2) = 30 + 10 = \boxed{40}$

29. $\int_1^e \frac{1}{x} dx = \ln x \Big|_1^e = \ln e - \ln 1 = 1 - 0 = \boxed{1}$

33. Area $= \int_0^{\pi/4} \sec^2 x dx = \tan x \Big|_0^{\pi/4} = \tan\frac{\pi}{4} - \tan 0 = 1 - 0 = 1$

Avg. y value $= \frac{\text{Area}}{\Delta x} = \frac{1}{\pi/4 - 0} = \frac{1}{\pi/4} = \boxed{\frac{4}{\pi}}$

37. $\int_0^1 \frac{1}{1+x^4}\,dx$

Min y value $= \frac{1}{1+1^4} = \frac{1}{1+1} = \frac{1}{2}$

Max y value $= \frac{1}{1+0^4} = \frac{1}{1+0} = 1$

Min area $=$ Min $y \times \Delta x = \frac{1}{2}(1-0) = \frac{1}{2} \cdot 1 = \frac{1}{2}$

Max area $=$ Max $y \times \Delta x = 1 \times (1-0) = 1 \cdot 1 = 1$

Therefore $\boxed{\frac{1}{2} \le \int_0^1 \frac{1}{1+x^4}\,dx \le 1}$

41. $1000\,m^3$ at $10\ m^3/min$

$1000\ m^3$ at $20\ m^3/min$

Amount = Rate × Time → Time $= \frac{\text{Amount}}{\text{Rate}}$

Time 1 $= \frac{1000\ \cancel{m^3}}{10\ \cancel{m^3}/min} = 100$ min

Time 2 $= \frac{1000\ \cancel{m^3}}{20\ \cancel{m^3}/min} = 50$ min

Avg. rate $= \frac{\text{Total Amount}}{\text{Total Time}} = \frac{2000\ m^3}{150\ min} = \boxed{13.333\ m^3/min}$