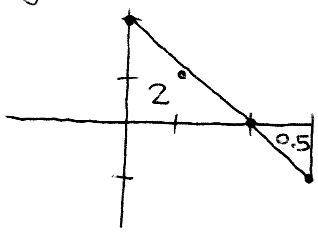
Section 6.4: 41–59 odd

41. $y = 2 - x = -x + 2$, $0 \le x \le 3$



$\frac{1}{2}\cdot 2\cdot 2 + \frac{1}{2}\cdot 1\cdot 1 = \frac{4}{2} + \frac{1}{2} = \boxed{\frac{5}{2}}$

Total Area → All counted as positive area

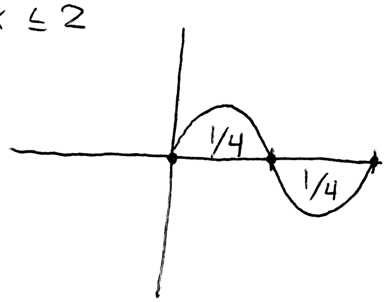
43. $y = x^3 - 3x^2 + 2x$, $0 \le x \le 2$

$x^3 - 3x^2 + 2x = 0$

$x(x^2 - 3x + 2) = 0$

$x(x-2)(x-1) = 0$

x-int : 0, 1, 2



Total Area $= \frac{1}{4} + \frac{1}{4} = \boxed{\frac{1}{2}}$

$$\int_0^1 (x^3 - 3x^2 + 2x)\,dx = \frac{1}{4}x^4 - x^3 + x^2 \Big|_0^1 = \left(\frac{1}{4} - 1 + 1\right) - (0 + 0 + 0) = \frac{1}{4}$$

$$\int_1^2 (x^3 - 3x^2 + 2x)\,dx = \frac{1}{4}x^4 - x^3 + x^2 \Big|_1^2 = (4 - 8 + 4) - \left(\frac{1}{4} \cancel{-1} \cancel{+1}\right) = 0 - \frac{1}{4} = -\frac{1}{4}$$

45. $$\int_0^1 x^2\,dx = \frac{1}{3}x^3 \Big|_0^1 = \frac{1}{3} - 0 = \frac{1}{3}$$

$$\int_1^2 (2-x)\,dx = 2x - \frac{1}{2}x^2 \Big|_1^2 = (4-2) - (2 - 1/2) = 2 - \frac{3}{2} = \frac{1}{2}$$

$$\frac{1}{3} + \frac{1}{2} = \frac{2}{6} + \frac{3}{6} = \boxed{\frac{5}{6}}$$

47. Shaded = Rectangle − Area Between $1 + \cos x$ and the x-axis

Rectangle = $bh = 2\pi$

White Area = $\int_0^\pi (1 + \cos x)\,dx = x + \sin x \Big|_0^\pi = (\pi + \sin\pi) - (0 + \sin 0) = \pi + 0 - 0 = \pi$

Shaded = $2\pi - \pi = \boxed{\pi}$

49. $\displaystyle\int_0^{10} \frac{1}{3+2\sin x}\,dx = \text{NINT}(1/(3+2\sin x), x, 0, 10) \approx \boxed{3.802}$

51. $y=\sqrt{8-2x^2}$

Find x-intercepts : $\sqrt{8-2x^2}=0 \to 8-2x^2=0 \to 2x^2=8 \to x^2=4 \to x=2,-2$

$\displaystyle\int_{-2}^{2} \sqrt{8-2x^2}\,dx = \text{NINT}(\sqrt{8-2x^2}, x, -2, 2) \approx \boxed{8.886}$

53. $\displaystyle\int_0^x e^{-t^2}dt = 0.6 \to$ Graph: $y_1 = \text{NINT}(e^{-x^2}, x, 0, x)$, $y_2 = 0.6$ Intersect at $\boxed{x=0.699}$

55. $\displaystyle\int_{-1}^{x} f(t)\,dt + k = \int_{2}^{x} f(t)\,dt \to k = \int_{2}^{x} f(t)\,dt - \int_{-1}^{x} f(t)\,dt = \int_{2}^{x} f(t)\,dt + \int_{x}^{-1} f(t)\,dt$

$\displaystyle k = \int_2^{-1} f(t)\,dt = \int_2^{-1} (t^2-3t+1)\,dt = \frac{1}{3}t^3 - \frac{3}{2}t^2 + t\Big|_2^{-1}$

$k = \left(-\frac{1}{3} - \frac{3}{2} - 1\right) - \left(\frac{8}{3} - 6 + 2\right) = \frac{-1}{3} - \frac{3}{2} - 1 - \frac{8}{3} + 6 - 2$

$k = -\cancel{\frac{9}{3}} - \frac{3}{2} + \cancel{3} \to \boxed{k = \frac{-3}{2}}$

57. $H(x)$ = area from 0 to x

a) $\displaystyle H(0) = \int_0^0 f(t)\,dt = \boxed{0}$ (haven't gained or lost area)

b) H increases when $H'(x) > 0$

$\displaystyle H'(x) = \frac{d}{dx}\int_0^x f(t)\,dt = f(x)$, so $H'(x) > 0$ when $f(x) > 0$, which is $\boxed{(0,6)}$

c) Concave up when $H''(x) > 0$

$H'(x) = f(x)$, so $H''(x) = f'(x)$

$H''(x) > 0$ when $f'(x) > 0$ (f is increasing), which is $\boxed{(9,12)}$

d) $\displaystyle H(12) = \int_0^{12} f(t)\,dt$ is $\boxed{\text{positive}}$ bc there is more + area than − area on [0,12]

57. e) $H'(x) = f(x)$

	0		6		12
$H'(x)$		+ + + +	0	− − − −	
	Rel Min		Max		Rel Min

Max at $\boxed{x=6}$ bc sign of $H'(x)$ changes from + to −

f) Rel mins at endpoints: $x=0$, $x=12$

$x=0$: 0 Area

$x=12$: + Area (explained in part 57 d)

0 area < + Area, so absolute min at $\boxed{x=0}$

59. $s = \text{position} = \int_0^t f(x)\,dx$

Velocity $= s' = f(t)$

acceleration $= s'' = f'(t)$

a) $v(3) = s'(3) = f(3) = \boxed{0}$

b) $a(3) = s''(3) = f'(3) =$ slope of f at $x=3 \rightarrow \boxed{\text{positive}}$

c) $s(3) = \int_0^3 f(x)\,dx$ = area from $t=0$ to $t=3$

$\Delta = -\frac{1}{2} \cdot 3 \cdot 6 = \boxed{-9 \text{ units}}$

d) $s(t)=0$ when amount of − area = amount of + area

$\boxed{t=6}$ bc area below x-axis cancels with area above x-axis

e) $a(t) = 0$ when $f'(t) = 0 \rightarrow$ slope of $f = 0$ at $\boxed{t=7s}$

f) Toward origin: when s is − and v is + or when s is + and v is −

Away from origin: s & v are both −, or s & v are both +

s = area, v = y value

	0		3		6		9
s		− − −		− − −	0	+ + +	
v		− − −		+ + +		+ + +	

Toward: $\boxed{(3,6)}$

Away: $\boxed{(0,3) \cup (6,9)}$

g) $s(9)$ is $\boxed{\text{positive}}$ bc there is more area above the x-axis (+) than below the x-axis (−) at $t=9$.