Section 7.1 - Day 2: 25-45 odd, 49, 61, 65, 75

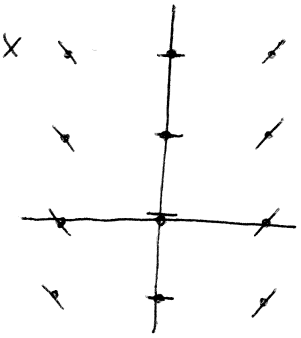
25. $\frac{dy}{dx} = (\sin x)^2$

Slope = 0 when $x = 0 \ (+k\pi)$, slope = + everywhere else → **Graph B**

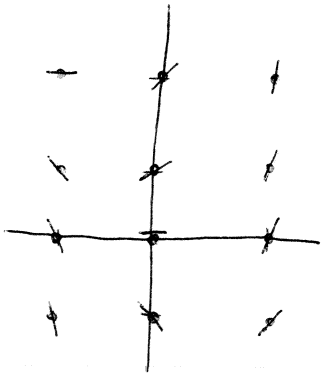
27. $\frac{dy}{dx} = (\cos x)^2$

Slope = 0 when $x = \frac{\pi}{2} \ (+k\pi)$, slope = + everywhere else → **Graph A**

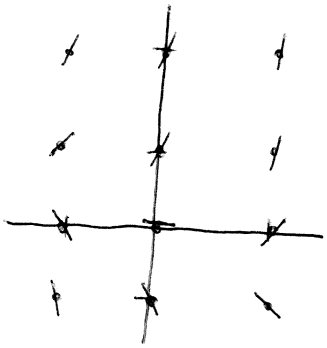
29. $\frac{dy}{dx} = x$



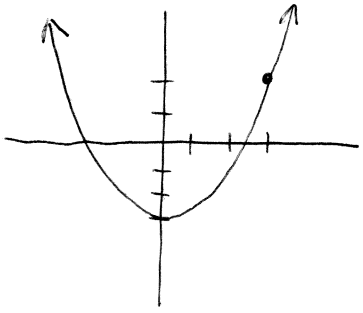
31. $\frac{dy}{dx} = 2x+y$



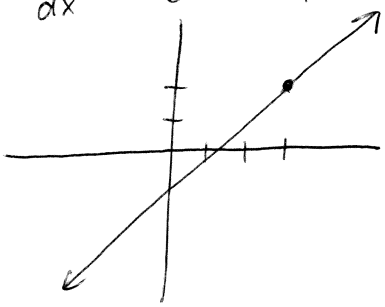
33. $\frac{dy}{dx} = x+2y$



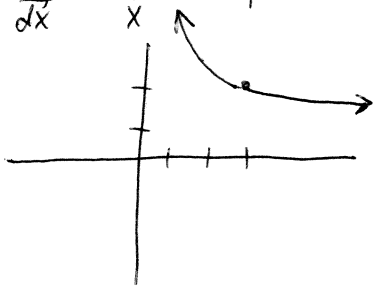
35. $\frac{dy}{dx} = x$ → **Graph C**



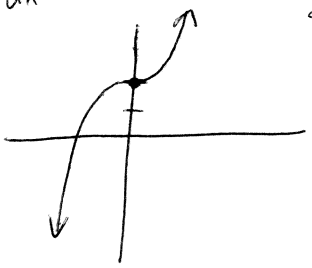
37. $\frac{dy}{dx} = x - y \rightarrow$ slope $= 0$ whenever $x = y \rightarrow$ **Graph A**



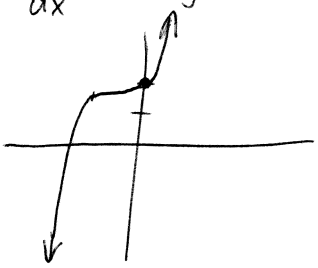
39. $\frac{dy}{dx} = \frac{-y}{x} \rightarrow$ slope $= 0$ when $y = 0$, undef. when $x = 0 \rightarrow$ **Graph B**



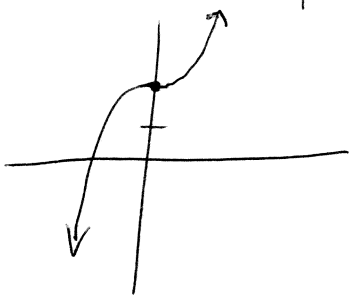
41. $\frac{dy}{dx} = \sqrt{x^2 - x + 1} \rightarrow$ same slope for same value of $x \rightarrow$ **Graph D**
& slope $= 1$ when $x = 0$



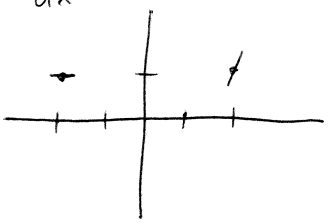
43. $\frac{dy}{dx} = |x + y| \rightarrow$ slope $= 0$ when $x = -y$ (quadrants 2 & 4) $\rightarrow$ **Graph C**



45. $\frac{dy}{dx} = |x| \rightarrow$ same slope for same value of x, all positive $\rightarrow$ **Graph B**
slope $= 0$ when $x = 0$

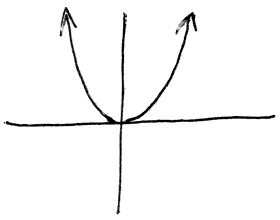


49. $\frac{dy}{dx} = 2y + x$ at $x = -2, y = 1 \rightarrow 2(1) + -2 = 2 - 2 = 0$ slope at $(-2,1) \rightarrow$ **Graph C**



At $(2,1)$

$2(1) + 2 = \boxed{4}$

61. 

$y = x^2$ has negative slopes in the second quadrant. However, the slope field shown has positive slopes in the second quadrant.

65. When slopes are perpendicular, their product is -1.

$e^{\frac{x-y}{2}} \cdot -e^{\frac{y-x}{2}} = -e^{\frac{1}{2}x - \frac{1}{2}y + \frac{1}{2}y - \frac{1}{2}x} = -e^0 = \boxed{-1}$

75. $\frac{dy}{dx} = x - \frac{1}{x^2} = x - x^{-2}$

a) $y = \frac{1}{2}x^2 + x^{-1} + C = \frac{1}{2}x^2 + \frac{1}{x} + C$ at $(1,2)$

$2 = \frac{1}{2} \cdot 1^2 + \frac{1}{1} + C \rightarrow 2 = \frac{1}{2} + 1 + C \rightarrow C = \frac{1}{2} \rightarrow \boxed{y = \frac{1}{2}x^2 + \frac{1}{x} + \frac{1}{2}}$

b) $y = \frac{1}{2}x^2 + \frac{1}{x} + C$ at $(-1,1)$

$1 = \frac{1}{2}(-1)^2 + \frac{1}{-1} + C \rightarrow 1 = \frac{1}{2} - 1 + C \rightarrow C = \frac{3}{2} \rightarrow \boxed{y = \frac{1}{2}x^2 + \frac{1}{x} + \frac{3}{2}}$

c) $y = \begin{cases} x^{-1} + \frac{1}{2}x^2 + C_1, & x < 0 \\ x^{-1} + \frac{1}{2}x^2 + C_2, & x > 0 \end{cases} \rightarrow \frac{dy}{dx} = \begin{cases} \frac{-1}{x^2} + x, & x < 0 \\ \frac{-1}{x^2} + x, & x > 0 \end{cases}$

$\frac{dy}{dx} = \frac{-1}{x^2} + x, \quad x \neq 0$ (not in domain)

d) C_1 when $x < 0 \rightarrow \boxed{C_1 = \frac{3}{2}}$ from part b

C_2 when $x > 0 \rightarrow \boxed{C_2 = \frac{1}{2}}$ from part a

e) $y(2) = -1$, $x > 0$ so C_2

$-1 = \frac{1}{2} \cdot 2^2 + \frac{1}{2} + C_2 \rightarrow -1 = 2 + \frac{1}{2} + C_2 \rightarrow \boxed{C_2 = \frac{-7}{2}}$

$y(-2) = 2$, $x < 0$ so C_1

$2 = \frac{1}{2}(-2)^2 + \frac{1}{-2} + C_1 \rightarrow 2 = 2 - \frac{1}{2} + C_1 \rightarrow \boxed{C_1 = \frac{1}{2}}$