Section 7.2: 1-69 e.o.o.

1. $\int (\cos x - 3x^2)\,dx = \boxed{\sin x - x^3 + C}$

5. $\int (3x^4 - 2x^{-3} + \sec^2 x)\,dx = \boxed{\frac{3}{5}x^5 + x^{-2} + \tan x + C}$

9. $\frac{d}{dx}\left(\frac{1}{2}e^{2x} + c\right) = \frac{1}{2}e^{2x} \cdot 2 + 0 = \boxed{e^{2x}}$

13. $f(u) = \sqrt{u} = u^{1/2}, \quad u = x^2, \quad x > 0$

$$\int f(u)\,du = \int u^{1/2}\,du = \frac{2}{3}u^{3/2} = \frac{2}{3}(x^2)^{3/2} = \boxed{\frac{2}{3}x^3 + C}$$

$$\int f(u)\,dx = \int \sqrt{x^2}\,dx = \int x\,dx = \boxed{\frac{1}{2}x^2 + C}$$

17. $\int \sin 3x\,dx$ $\quad u = 3x$, $du = 3\,dx$, $dx = \frac{du}{3}$

$$\int \sin u \cdot \frac{du}{3} = \frac{1}{3}\int \sin u\,du = -\frac{1}{3}\cos u + C = \boxed{-\frac{1}{3}\cos 3x + C}$$

$$\frac{d}{dx}\left(-\frac{1}{3}\cos 3x + C\right) = \cancel{\frac{1}{3}}\sin 3x \cdot \cancel{3} + 0 = \sin 3x \checkmark$$

21. $\int \frac{1}{x^2+9}\,dx$ $\quad u = \frac{1}{3}x,\ x = 3u,\ x^2 = 9u^2$, $du = \frac{1}{3}\,dx$, $dx = 3\,du$

$$\int \frac{1}{9u^2+9} \cdot 3\,du = 3\int \frac{1}{9(u^2+1)}\,du = \frac{3}{9}\int \frac{1}{u^2+1}\,du = \frac{1}{3}\tan^{-1}u + C = \boxed{\frac{1}{3}\tan^{-1}\left(\frac{x}{3}\right) + C}$$

$$\frac{d}{dx}\left(\frac{1}{3}\tan^{-1}\left(\frac{x}{3}\right) + C\right) = \frac{1}{3} \cdot \frac{1}{\left(\frac{1}{3}x\right)^2 + 1} \cdot \frac{1}{3} + 0 = \frac{1}{9} \cdot \frac{1}{\frac{1}{9}x^2 + 1} = \frac{1}{x^2+9} \checkmark$$

25. $\int \frac{1}{(1-x)^2}\,dx$ $\quad u = 1-x$, $du = -dx$, $dx = -du$

$$\int \frac{1}{u^2} \cdot -du = \int -u^{-2}\,du = u^{-1} + C = \frac{1}{u} + C = \boxed{\frac{1}{1-x} + C}$$

29. $\int \tan(4x+2)dx = \int \frac{\sin(4x+2)}{\cos(4x+2)} dx$

$u = \cos(4x+2)$
$du = -4\sin(4x+2)dx$
$dx = \frac{du}{-4\sin(4x+2)}$

$$\int \frac{\cancel{\sin(4x+2)}}{u} \cdot \frac{du}{-4\cancel{\sin(4x+2)}} = \frac{-1}{4}\int \frac{1}{u}\,du = \frac{-1}{4}\ln|u| + C = \boxed{-\frac{1}{4}\ln|\cos(4x+2)| + C}$$

33. $\int \frac{(\ln x)^6}{x} dx$

$u = \ln x$
$du = \frac{1}{x} dx$
$dx = x\,du$

$$\int \frac{u^6}{\cancel{x}} \cdot \cancel{x}\,du = \int u^6 du = \frac{1}{7}u^7 + C = \boxed{\frac{1}{7}(\ln x)^7 + C}$$

37. $\int \frac{\sin(2t+1)}{\cos^2(2t+1)} dt$

$u = \cos(2t+1)$
$du = -2\sin(2t+1)dt$
$dt = \frac{du}{-2\sin(2t+1)}$

$$\int \frac{\cancel{\sin(2t+1)}}{u^2} \cdot \frac{du}{-2\cancel{\sin(2t+1)}} = \frac{-1}{2}\int u^{-2}du = \frac{1}{2}u^{-1} + C = \frac{1}{2u} + C = \boxed{\frac{1}{2\cos(2t+1)} + C \text{ or } \frac{1}{2}\sec(2t+1) + C}$$

41. $\int \frac{x}{x^2+1} dx$

$u = x^2 + 1$
$du = 2x\,dx$
$dx = \frac{du}{2x}$

$$\int \frac{\cancel{x}}{u} \cdot \frac{du}{2\cancel{x}} = \frac{1}{2}\int \frac{1}{u}\,du = \frac{1}{2}\ln|u| + C = \boxed{\frac{1}{2}\ln|x^2+1| + C}$$

45. $\int \sec x\,dx = \int \frac{\sec x}{1}\left(\frac{\sec x + \tan x}{\sec x + \tan x}\right) dx = \int \frac{\sec^2 x + \sec x \tan x}{\sec x + \tan x} dx$

$u = \sec x + \tan x$
$du = (\sec x \tan x + \sec^2 x)dx$
$dx = \frac{du}{\sec x \tan x + \sec^2 x}$

$$\int \frac{\cancel{\sec^2 x + \sec x \tan x}}{u} \cdot \frac{du}{\cancel{\sec x \tan x + \sec^2 x}} = \int \frac{1}{u}\,du = \ln|u| + C = \boxed{\ln|\sec x + \tan x| + C}$$

49. $\int 2\sin^2 x\,dx$ $\qquad \cos 2x = 1 - 2\sin^2 x \rightarrow 2\sin^2 x = 1 - \cos 2x$

$$\int (1-\cos 2x)\,dx = \boxed{x - \tfrac{1}{2}\sin 2x + C}$$

53. $\int_0^3 (y+1)^{1/2}\,dy$ $\qquad U = y+1$, $dU = dy$

$$\int U^{1/2}\cdot dU = \frac{2}{3}U^{3/2} = \frac{2}{3}(y+1)^{3/2}\Big|_0^3 = \frac{2}{3}(4)^{3/2} - \frac{2}{3}(1)^{3/2} = \frac{2}{3}(8-1) = \frac{2}{3}\cdot 7 = \boxed{\frac{14}{3}}$$

57. $\int_0^1 \frac{10\sqrt{\theta}}{(1+\theta^{3/2})^2}\,d\theta$ $\qquad U = 1+\theta^{3/2}$, $dU = \frac{3}{2}\theta^{1/2}\,d\theta$, $d\theta = \frac{dU}{\frac{3}{2}\sqrt{\theta}} = \frac{2\,dU}{3\sqrt{\theta}}$

$$\int \frac{10\cancel{\sqrt{\theta}}}{U^2}\cdot\frac{2\,dU}{3\cancel{\sqrt{\theta}}} = \frac{20}{3}\int U^{-2}\,dU = \frac{-20}{3}U^{-1} = \frac{-20}{3U} = \frac{-20}{3+3\theta^{3/2}}\Big|_0^1$$

$$\frac{-20}{3+3\cdot 1} + \frac{20}{3+0} = \frac{-20}{6} + \frac{20}{3} = \frac{-20}{6} + \frac{40}{6} = \frac{20}{6} = \boxed{\frac{10}{3}}$$

61. $\int_0^7 \frac{1}{x+2}\,dx$ $\qquad U = x+2$, $dU = dx$

$$\int \frac{1}{U}\cdot dU = \ln|U| = \ln|x+2|\Big|_0^7 = \boxed{\ln 9 - \ln 2 \approx 1.504}$$

65. $\int_{-1}^3 \frac{x}{x^2+1}\,dx$ $\qquad U = x^2+1$, $dU = 2x\,dx$, $dx = \frac{dU}{2x}$

$$\int \frac{\cancel{x}}{U}\cdot\frac{dU}{2\cancel{x}} = \frac{1}{2}\int\frac{1}{U}\,dU = \frac{1}{2}\ln|U| = \frac{1}{2}\ln(x^2+1)\Big|_{-1}^3 = \frac{1}{2}\ln 10 - \frac{1}{2}\ln 2 = \frac{1}{2}\ln 5 = \boxed{\ln\sqrt{5} \approx 0.805}$$

69. $y = \ln \left| \frac{\cos 3}{\cos x} \right| + 5$

$y = \ln|\cos 3| - \ln|\cos x| + 5$

$\frac{dy}{dx} = \frac{1}{\cos 3} \cdot 0 - \frac{1}{\cos x}(-\sin x) + 0 = \cancel{0} + \frac{\sin x}{\cos x} + \cancel{0} = \frac{\sin x}{\cos x} = \boxed{\tan x}$

$y(3) = \ln \left| \frac{\cos 3}{\cos 3} \right| + 5 = \ln(1) + 5 = 0 + 5 = \boxed{5}$