Integration by Substitution (Section 7.2)

* We can use substitution to make integration easier!

ex: $\int \cos(3x-5)\,dx$

$u = 3x-5$

$du = 3dx/3$

$\frac{1}{3}du = dx$

$= \int \cos(u)\cdot\frac{1}{3}du$

$= \frac{1}{3}\sin u + C$

$\boxed{= \frac{1}{3}\sin(3x-5)+C}$

ex: $\boxed{\int \tan x\,dx}$

$= \int \frac{\sin x}{\cos x}dx$

$u = \cos x$

$du = -\sin x\,dx$

$-du = \sin x\,dx$

$= \int \frac{1}{\cos x}\cdot \sin x\,dx$

$= \int \frac{1}{u}\cdot -du$

$= -\ln|u| + C$

$\boxed{= -\ln|\cos x| + C \text{ OR } \ln|\sec x| + C}$

** $\int f(g(x))\cdot g'(x)\,dx = \int f(u)\,du$

$= F(u) + C$

$= F(g(x)) + C$

Let $u = g(x)$

$du = g'(x)\,dx$

ex: $\int 8(y^4+4y^2+1)^2(y^3+2y)\,dy$

$2 \cdot 4$

$u = y^4+4y^2+1$

$du = (4y^3+8y)\,dy$

$du = 4(y^3+2y)\,dy$

$= \int 2u^2\,du$

$= \frac{2}{3}u^3 + C$

$\boxed{= \frac{2}{3}(y^4+4y^2+1)^3 + C}$

ex: $\int \frac{6\cos t}{(2+\sin t)^2}dt$

$u = 2+\sin t$

$du = \cos t\,dt$

$= \int \frac{6}{u^2}du = \int 6u^{-2}du$

$= -6u^{-1} + C$

$\boxed{= -6(2+\sin t)^{-1} + C}$

#59 $\int \sqrt{t^5+2t}\,(5t^4+2)\,dt$

$u = t^5+2t$

$du = (5t^4+2)\,dt$

$= \int u^{1/2}du$

$= \frac{2}{3}u^{3/2} + C$

$\boxed{= \frac{2}{3}(t^5+2t)^{3/2} + C}$

ex: $\int \tan^7\left(\frac{x}{2}\right)\cdot \sec^2\left(\frac{x}{2}\right)dx$

$u = \tan\left(\frac{x}{2}\right)$

$du = \sec^2\left(\frac{x}{2}\right)dx \cdot \frac{1}{2}$

$2du = \sec^2\left(\frac{x}{2}\right)dx$

$= \int u^7 \cdot 2\,du$

$= \frac{2u^8}{8} + C$

$\boxed{= \frac{1}{4}\tan^8\left(\frac{x}{2}\right) + C}$

ex: $\int_{-\pi}^{\pi} \frac{\cos x}{\sqrt{4+3\sin x}}\,dx$

* If you use a "u-sub" on a definite integral, you MUST change the limits of integration to u-values!

$u = 4+3\sin x$

$du = 3\cos x\,dx$

$\frac{1}{3}du = \cos x\,dx$

$u(\pi) = 4+3\sin(\pi) = 4$

$u(-\pi) = 4+3\sin(-\pi) = 4$

$\int_4^4 \frac{1}{\sqrt{u}}\cdot\frac{1}{3}du \boxed{= 0}$

* If it was $\int_{u=4}^{u=25} \frac{1}{3}u^{-1/2}du$

$= \frac{2}{3}u^{1/2}\Big|_4^{25}$

$= \frac{2}{3}\left[(25)^{1/2} - (4)^{1/2}\right]$

$= \frac{2}{3}(5-2) = \boxed{2}$

#60 $\int_0^{\pi/6} \cos^{-3}(2\theta)\cdot\sin(2\theta)\,d\theta$

$u = \cos(2\theta)$

$du = -\sin(2\theta)\cdot 2\,d\theta$

$-\frac{1}{2}du = \sin(2\theta)\,d\theta$

$u(\pi/6) = \cos(2\pi/6) = 1/2$

$u(0) = \cos(0) = 1$

$= \int_1^{1/2} u^{-3}\cdot\frac{-1}{2}du$

$= \frac{-1}{2}\cdot\frac{u^{-2}}{-2}\Big|_1^{1/2}$

$= \frac{1}{4}\left[(1/2)^{-2} - (1)^{-2}\right]$

$= \frac{1}{4}(4-1) = \boxed{\frac{3}{4}}$