Section 7.3: 1-33 e.o.o.

1. $\int x\sin x\,dx$ $\quad u = x$ $\quad dv = \sin x\,dx$ $\quad du = dx$ $\quad v = -\cos x$

$$uv - \int v\,du = -x\cos x - \int -\cos x\,dx = -x\cos x - -\sin x + C = \boxed{-x\cos x + \sin x + C}$$

5. $\int x^2\cos x\,dx$ $\quad u = x^2$ $\quad dv = \cos x\,dx$ $\quad du = 2x\,dx$ $\quad v = \sin x$

$$x^2\sin x - \int 2x\sin x\,dx \quad u = 2x \quad dv = \sin x\,dx \quad du = 2dx \quad v = -\cos x$$

$$x^2\sin x - \left(-2x\cos x - \int -2\cos x\,dx\right) = x^2\sin x + 2x\cos x + \int -2\cos x\,dx$$

$$\boxed{x^2\sin x + 2x\cos x - 2\sin x + C}$$

9. $\int y\ln y\,dy$ $\quad u = \ln y$ $\quad dv = y\,dy$ $\quad du = \frac{1}{y}dy$ $\quad v = \frac{1}{2}y^2$

$$\frac{1}{2}y^2\ln y - \int \frac{1}{2}y^2 \cdot \frac{1}{y}\,dy = \frac{1}{2}y^2\ln y - \int \frac{1}{2}y\,dy = \boxed{\frac{1}{2}y^2\ln y - \frac{1}{4}y^2 + C}$$

13. $\int x\sec^2 x\,dx$ $\quad u = x$ $\quad dv = \sec^2 x\,dx$ $\quad du = dx$ $\quad v = \tan x$

$$v = x\tan x - \int \tan x\,dx = x\tan x + \ln|\cos x| + C$$

$v = 1$ when $x = 0$

$$1 = 0\tan 0 + \ln|\cos 0| + C \to 1 = 0 + 0 + C \to C = 1 \to \boxed{v = x\tan x + \ln|\cos x| + 1}$$

17. $\int e^x \sin x\,dx$ $\quad u = e^x$, $du = e^x dx$ $\quad dv = \sin x\,dx$, $v = -\cos x$ $\quad u = e^x$, $du = e^x dx$ $\quad dv = \cos x\,dx$, $v = \sin x$

$$\int e^x \sin x\,dx = -e^x \cos x - \int -e^x \cos x\,dx = -e^x \cos x + \int e^x \cos x\,dx$$

$$\int e^x \sin x\,dx = -e^x \cos x + e^x \sin x - \int e^x \sin x\,dx$$

$$2\int e^x \sin x\,dx = -e^x \cos x + e^x \sin x$$

$$\int e^x \sin x\,dx = \boxed{-\tfrac{1}{2} e^x \cos x + \tfrac{1}{2} e^x \sin x + C}$$

21. $\int x^4 e^{-x}\,dx$

u & deriv.		dv & antideriv.
x^4	+	e^{-x}
$4x^3$	−	$-e^{-x}$
$12x^2$	+	e^{-x}
$24x$	−	$-e^{-x}$
24	+	e^{-x}
0		$-e^{-x}$

$$\boxed{-x^4 e^{-x} - 4x^3 e^{-x} - 12x^2 e^{-x} - 24x e^{-x} - 24 e^{-x} + C}$$

25. $\int_0^{\pi/2} x^2 \sin 2x\,dx$

u & deriv.		dv & antideriv.
x^2	+	$\sin 2x$
$2x$	−	$-\frac{1}{2}\cos 2x$
2	+	$-\frac{1}{4}\sin 2x$
0		$\frac{1}{8}\cos 2x$

$$\left[-\tfrac{1}{2}x^2 \cos 2x + \tfrac{1}{2} x \sin 2x + \tfrac{1}{4}\cos 2x\right]\Big|_0^{\pi/2}$$

$$\left(-\tfrac{1}{2}\left(\tfrac{\pi}{2}\right)^2(-1) + 0 + \tfrac{1}{4}(-1)\right) - \left(0 + 0 + \tfrac{1}{4}(1)\right)$$

$$\frac{\pi^2}{8} - \frac{1}{4} - \frac{1}{4} = \boxed{\frac{\pi^2}{8} - \frac{1}{2}}$$

29. $\int x^2 e^{4x}\,dx$

u & deriv.		dV & antideriv.
x^2	+	e^{4x}
$2x$	−	$\frac{1}{4}e^{4x}$
2	+	$\frac{1}{16}e^{4x}$
0		$\frac{1}{64}e^{4x}$

$$\boxed{\frac{1}{4}x^2e^{4x} - \frac{1}{8}xe^{4x} + \frac{1}{32}e^{4x} + C}$$

33. $\int x\sin x\,dx$

u & deriv.		dV & antideriv.
x	+	$\sin x$
1	−	$-\cos x$
0		$-\sin x$

$-x\cos x + \sin x$

a) $(-x\cos x + \sin x)\Big|_0^{\pi} = (-\pi(-1)+0) - (0+0) = \boxed{\pi}$

b) $(-x\cos x + \sin x)\Big|_{\pi}^{2\pi} = (-2\pi(1)+0) - (-\pi(-1)+0) = -2\pi - \pi = \boxed{-3\pi}$ (below axis)

3π total area

c) $(-x\cos x + \sin x)\Big|_0^{2\pi} = \pi - 3\pi = \boxed{-2\pi \text{ (below axis)}}$

$\pi + 3\pi = \boxed{4\pi \text{ (total area)}}$