

Integration by Parts (Section 7.3)

* $\int u dv = u \cdot v - \int v du$

where "u" is usually a polynomial and "dv" is the rest.

* This process can be done more than once!

ex: $\int x e^{-x} dx$

$u = x$ $dv = e^{-x} dx$
 $du = dx$ $v = -e^{-x}$

$u \cdot v - \int v du$
 $= -x \cdot e^{-x} + \int e^{-x} dx$

$= -x e^{-x} - e^{-x} + C$

ex: $\int x \sec^2 x dx$

$u = x$ $dv = \sec^2 x dx$
 $du = dx$ $v = \tan x$

$u \cdot v - \int v du$
 $= x \cdot \tan x - \int \tan x dx$

$= x \tan x + \ln |\cos x| + C$

$= -\ln |\cos x| + C$
 $\text{OR } \ln |\sec x| + C$

#5 ex: $\int x^2 \cos x dx$

$u = x^2$ $dv = \cos x dx$
 $du = 2x dx$ $v = \sin x$

$x^2 \sin x - \int \sin x \cdot 2x dx$

$u_2 = 2x$ $dv = \sin x dx$
 $du = 2 dx$ $v = -\cos x$

$= x^2 \sin x + \int +2x \cos x - \int +2 \cos x dx$

$= x^2 \sin x + 2x \cos x - 2 \sin x + C$

$\int x^2 \cos x dx$

<u>f(x): derive</u>		<u>g(x): integrate</u>
x^2	$(+1)$	$\cos x$
$2x$	(-1)	$\sin x$
2	$(+1)$	$-\cos x$
0		$-\sin x$

↑ Alternate signs

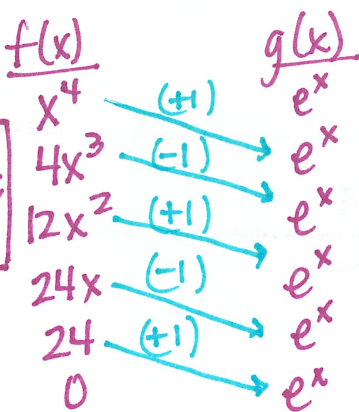
$x^2 \sin x + 2x \cos x - 2 \sin x + C$

← same

* Tabular Integration: If f(x) can be differentiated repeatedly to become zero (polynomial) and g(x) can be integrated repeatedly w/o difficulty (sin, cos, e^x...) then we usually use tabular integration.

ex: $\int x^4 e^x dx$

$-e^x [x^4 - 4x^3 + 12x^2 - 24x + 24] + C$



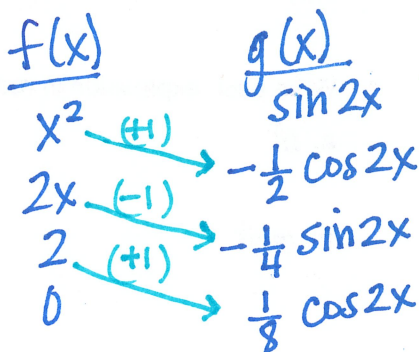
#25 $\int_0^{\pi/2} x^2 \sin 2x dx$

$= -\frac{x^2}{2} \cos 2x + \frac{x}{2} \sin 2x + \frac{1}{4} \cos 2x \Big|_0^{\pi/2}$

$= -\frac{(\pi/2)^2}{2} \cos(\pi) + \frac{(\pi/2)}{2} \sin(\pi) + \frac{1}{4} \cos(\pi) - [0 + 0 + \frac{1}{4} \cos(0)]$

$= \frac{\pi^2}{8} - \frac{1}{4} - \frac{1}{4}$

$= \frac{\pi^2}{8} - \frac{1}{2}$



ex: $\int e^x \cos x dx = e^x \sin x - \int e^x \sin x dx$
 $= e^x \sin x + [e^x \cos x - \int e^x \cos x dx]$
 $= e^x \sin x + e^x \cos x - \int e^x \cos x dx$

$u = e^x \quad dv = \cos x dx$
 $du = e^x dx \quad v = \sin x$

$u_2 = e^x \quad dv = \sin x dx$
 $du = e^x dx \quad v = -\cos x$

$\int e^x \cos x dx = e^x \sin x + e^x \cos x - \int e^x \cos x dx + \int e^x \cos x dx$

$2 \int e^x \cos x dx = \frac{e^x \sin x + e^x \cos x}{2}$

$\int e^x \cos x dx = \frac{1}{2} (e^x \sin x + e^x \cos x)$