Integration by Parts (Section 7.3)

* $\int u\,dv = u\cdot v - \int v\,du$ where "u" is usually a polynomial and "dv" is the rest.

* This process can be done more than once!

ex: $\int x e^{-x}\,dx$

$u = x \qquad dv = e^{-x}dx$

$du = dx \qquad v = -e^{-x}$

$u\cdot v - \int v\,du$

$= -x\cdot e^{-x} + \int + e^{-x}dx$

$\boxed{= -xe^{-x} - e^{-x} + C}$

ex: $\int x\sec^2 x\,dx$

$u = x \qquad dv = \sec^2 x\,dx$

$du = dx \qquad v = \tan x$

$u\cdot v - \int v\,du$

$= x\cdot\tan x - \boxed{\int \tan x\,dx = -\ln|\cos x| + c \text{ OR } \ln|\sec x| + c}$

$\boxed{= x\tan x + \ln|\cos x| + c}$

#5 ex: $\int x^2\cos x\,dx$

$u = x^2 \qquad dv = \cos x\,dx$

$du = 2x\,dx \qquad v = \sin x$

$x^2\sin x - \int \underline{\sin x\cdot 2x\,dx}$

$u_2 = 2x \qquad dv = \sin x\,dx$

$du = 2dx \qquad v = -\cos x$

$= x^2\sin x + \left[+2x\cos x - \int +2\cos x\,dx\right]$

$\boxed{= x^2\sin x + 2x\cos x - 2\sin x + C}$

same

$\int x^2\cos x\,dx$

f(x): derive		g(x): integrate
x^2	$(+1)$	$\cos x$
$2x$	(-1)	$\sin x$
2	$(+1)$	$-\cos x$
0		$-\sin x$

Alternate signs

$\boxed{x^2\sin x + 2x\cos x - 2\sin x + C}$

* Tabular Integration: If f(x) can be differentiated repeatedly to become zero (polynomial) and g(x) can be integrated repeatedly w/o difficulty ($\sin, \cos, e^x\ldots$) then we usually use tabular integration.

ex: $\int x^4 e^x \, dx$

$$\boxed{= e^x\left[x^4 - 4x^3 + 12x^2 - 24x + 24\right] + C}$$

$f(x)$		$g(x)$
x^4		e^x
$4x^3$	$(+1)$	e^x
$12x^2$	(-1)	e^x
$24x$	$(+1)$	e^x
24	(-1)	e^x
0	$(+1)$	e^x

#25 $\int_0^{\pi/2} x^2 \sin 2x \, dx$

$$= -\frac{x^2}{2}\cos 2x + \frac{x}{2}\sin 2x + \frac{1}{4}\cos 2x \Big|_0^{\pi/2}$$

$$= -\frac{(\pi/2)^2}{2}\overset{-1}{\cos(\pi)} + \frac{(\pi/2)}{2}\overset{0}{\sin(\pi)} + \frac{1}{4}\overset{-1}{\cos(\pi)}$$

$$- \left[0 + 0 + \frac{1}{4}\overset{+1}{\cos(0)}\right]$$

$$= \frac{\pi^2}{8} - \frac{1}{4} - \frac{1}{4}$$

$$\boxed{= \frac{\pi^2}{8} - \frac{1}{2}}$$

$f(x)$		$g(x)$
x^2		$\sin 2x$
$2x$	$(+1)$	$-\frac{1}{2}\cos 2x$
2	(-1)	$-\frac{1}{4}\sin 2x$
0	$(+1)$	$\frac{1}{8}\cos 2x$

ex: $\int e^x \cos x \, dx = e^x \sin x - \int e^x \sin x \, dx$

$u = e^x \qquad dv = \cos x \, dx$

$du = e^x dx \qquad v = \sin x$

$u_2 = e^x \qquad dv = \sin x \, dx$

$du = e^x dx \qquad v = -\cos x$

$$= e^x \sin x + \left[+e^x \cos x - \int +e^x \cos x \, dx\right]$$

$$= e^x \sin x + e^x \cos x - \int e^x \cos x \, dx$$

$$\int e^x \cos x \, dx = e^x \sin x + e^x \cos x - \cancel{\int e^x \cos x \, dx}$$

$$+\int e^x \cos x \, dx \qquad\qquad + \cancel{\int e^x \cos x \, dx}$$

$$\frac{2\int e^x \cos x \, dx}{2} = \frac{e^x \sin x + e^x \cos x}{2}$$

$$\boxed{\int e^x \cos x \, dx = \frac{1}{2}\left(e^x \sin x + e^x \cos x\right)}$$