Section 7.4 : 1-15 odd, 16-19 all, 21-29 odd

1. $\frac{dy}{dx} = \frac{x}{y}$ and $y = 2$ when $x = 1$

$\int y\,dy = \int x\,dx$

$\frac{1}{2}y^2 = \frac{1}{2}x^2 + C$

$\frac{1}{2}\cdot 2^2 = \frac{1}{2}\cdot 1^2 + C \rightarrow \frac{4}{2} = \frac{1}{2} + C \rightarrow C = \frac{3}{2}$

$\frac{1}{2}y^2 = \frac{1}{2}x^2 + \frac{3}{2}$

$y^2 = x^2 + 3$

$\boxed{y = \sqrt{x^2+3}}$

Domain: $\boxed{(-\infty, \infty)}$ bc x^2+3 is always positive

3. $\frac{dy}{dx} = \frac{y}{x}$ and $y = 2$ when $x = 2$

$\int \frac{1}{y}\,dy = \int \frac{1}{x}\,dx$

$\ln|y| = \ln|x| + C$

$\ln 2 = \ln 2 + C \rightarrow C = 0$

$\cancel{e^{\ln}}|y| = \cancel{e^{\ln}}|x|$

$|y| = |x|$

$\boxed{y = x}$

Domain: $\boxed{(0, \infty)}$ bc ln can only take in positive values

5. $\frac{dy}{dx} = (y+5)(x+2)$ and $y = 1$ when $x = 0$

$\int \frac{1}{y+5}\,dy = \int (x+2)\,dx$

$\ln|y+5| = \frac{1}{2}x^2 + 2x + C$

$\ln 6 = 0 + 0 + C \rightarrow C = \ln 6$

$e^{\ln|y+5|} = e^{\frac{1}{2}x^2 + 2x + \ln 6}$

$|y+5| = e^{\frac{1}{2}x^2 + 2x + \ln 6} = e^{\frac{1}{2}x^2+2x}\cdot \cancel{e^{\ln 6}} = 6e^{\frac{1}{2}x^2 + 2x}$

$y + 5 = 6e^{\frac{1}{2}x^2 + 2x}$

$\boxed{y = 6e^{\frac{1}{2}x^2+2x} - 5}$ Domain: $\boxed{(-\infty, \infty)}$ bc e functions can have any x value

7. $\frac{dy}{dx} = (\cos x) \cdot e^{y+\sin x}$ and $y=0$ when $x=0$

$\frac{dy}{dx} = \cos x \cdot e^{y} \cdot e^{\sin x}$

$\frac{1}{e^y} dy = \cos x \cdot e^{\sin x} dx$

$\int e^{-y} dy = \int \cos x \cdot e^{\sin x} dx$

$-e^{-y} = e^{\sin x} + C$

$-e^{0} = e^{0} + C \rightarrow -1 = 1 + C \rightarrow C = -2$

$-e^{-y} = e^{\sin x} - 2$

$e^{-y} = -(e^{\sin x} - 2)$

$e^{-y} = 2 - e^{\sin x}$

$-y \cdot \cancel{\ln e} = \ln(2 - e^{\sin x})$

$-y = \ln(2 - e^{\sin x})$

$\boxed{y = -\ln(2 - e^{\sin x})}$

Domain : Where $2 - e^{\sin x} > 0$, so $\boxed{e^{\sin x} < 2}$ because ln only takes in positives.

9. $\frac{dy}{dx} = -2xy^2$ and $y = 0.25$ when $x = 1$

$\frac{1}{y^2} dy = -2x\, dx$

$\int y^{-2} dy = \int -2x\, dx$

$-y^{-1} = -x^2 + C$

$-\frac{1}{y} = -x^2 + C$

$-\frac{1}{1/4} = -1^2 + C \rightarrow -4 = -1 + C \rightarrow C = -3$

$-\frac{1}{y} = -x^2 - 3$

$\frac{1}{y} = \frac{x^2 + 3}{1}$

$\boxed{y = \frac{1}{x^2+3}}$

Domain: $\boxed{(-\infty, \infty)}$ bc x^2+3 is always positive, never $= 0$

11. $\frac{dy}{dt} = ky$, $k = 1.5$, $y(0) = 100$

$\int \frac{1}{y}\, dy = \int k\, dt$

$e^{\ln|y|} = e^{kt + C}$

$|y| = e^{kt+C} = e^{kt} \cdot \underbrace{e^{C}}_{y_0}$

$y = y_0 e^{kt}$

$\boxed{y = 100e^{1.5t}}$

13. $y(0) = 50$, $y(5) = 100$

$y = y_0 e^{kt}$

$y = 50e^{kt}$

$100 = 50e^{5k}$

$2 = e^{5k}$

$\ln 2 = 5k \cdot \cancel{\ln e}$

$k = \frac{\ln 2}{5} \approx 0.139$

$\boxed{y = 50e^{0.139t}}$

5. $A_0 = \$1{,}000$, $r = 8.6\% = 0.086$

$A = A_0 e^{rt}$

Doubling time:

$2000 = 1000e^{0.086t}$

$2 = e^{0.086t}$

$\ln 2 = 0.086t \cdot \cancel{\ln e}$

$t = \frac{\ln 2}{0.086} = \boxed{8.060 \text{ yr}}$

Amount in 30 yr:

$A = 1000e^{0.086(30)}$

$A = \boxed{\$13{,}197.14}$

6. $A_0 = \$2{,}000$, Doubling time = 15 yr

Annual rate:

$4000 = 2000e^{15r}$

$2 = e^{15r}$

$\ln 2 = 15r \cdot \cancel{\ln e}$

$r = \frac{\ln 2}{15} = 0.04621 = \boxed{4.621\%}$

Amount in 30 yr:

$A = 2000e^{0.04621(30)}$

$A = \boxed{\$8{,}000.00}$

17. $r = 5.25\% = 0.0525$, \$2,898.44 in 30 yr

Initial deposit:

$$2898.44 = A_0 e^{0.0525(30)}$$

$$A_0 = \frac{2898.44}{e^{0.0525(30)}} = \boxed{\$600.00}$$

Doubling time:

$$1200 = 600e^{0.0525t}$$

$$2 = e^{0.0525t}$$

$$\ln 2 = 0.0525t \cdot \cancel{\ln e}$$

$$t = \frac{\ln 2}{0.0525} = \boxed{13.203 \text{ yr}}$$

18. $A_0 = \$1,200$, \$10,405.37 in 30 yr

Rate:

$$10,405.37 = 1200e^{30r}$$

$$8.671 = e^{30r}$$

$$\ln 8.671 = 30r \cdot \cancel{\ln e}$$

$$r = \frac{\ln 8.671}{30} = 0.07200 = \boxed{7.200\%}$$

Doubling time:

$$2400 = 1200e^{0.072t}$$

$$2 = e^{0.072t}$$

$$\ln 2 = 0.072t \cdot \cancel{\ln e}$$

$$t = \frac{\ln 2}{0.072} = \boxed{9.627 \text{ yr}}$$

19. $A_0 = \$2,000$, $r = 4.75\% = 0.0475$ $\qquad A = A_0\left(1 + \frac{r}{n}\right)^{nt}$

a) Annually $\rightarrow n = 1$

$$4000 = 2000\left(1 + \frac{0.0475}{1}\right)^{1 \cdot t}$$

$$2 = 1.0475^t$$

$$\ln 2 = t \cdot \ln 1.0475$$

$$t = \frac{\ln 2}{\ln 1.0475} = \boxed{14.936 \text{ yr}}$$

b) Monthly $\rightarrow n = 12$

$$4000 = 2000\left(1 + \frac{0.0475}{12}\right)^{12t}$$

$$2 = 1.00396^{12t}$$

$$\ln 2 = 12t \cdot \ln 1.00396$$

$$t = \frac{\ln 2}{12 \ln 1.00396} = \boxed{14.621 \text{ yr}}$$

c) Quarterly $\rightarrow n = 4$

$$4000 = 2000\left(1 + \frac{0.0475}{4}\right)^{4t}$$

$$2 = 1.0119^{4t}$$

$$\ln 2 = 4t \cdot \ln 1.0119$$

$$t = \frac{\ln 2}{4 \ln 1.0119} = \boxed{14.679 \text{ yr}}$$

d) Continuously

$$A = A_0 e^{rt}$$

$$4000 = 2000e^{0.0475t}$$

$$2 = e^{0.0475t}$$

$$\ln 2 = 0.0475t \cdot \cancel{\ln e}$$

$$t = \frac{\ln 2}{0.0475} = \boxed{14.593 \text{ yr}}$$

21. $\frac{dy}{dt} = -0.0077y = ky \rightarrow k = -0.0077$

$y = y_0 e^{kt} \rightarrow y = y_0 e^{-0.0077t}$

$1 = 2e^{-0.0077t}$

$0.5 = e^{-0.0077t}$

$\ln 0.5 = -0.0077t \cdot \cancel{\ln e}$

$t = \frac{\ln 0.5}{-0.0077} = \boxed{90.019 \text{ yr}}$

23. $y_0 = 1$, doubles every 0.5 hr

$y = y_0 e^{kt}$

a) $2 = 1e^{k(0.5)}$

$2 = e^{0.5k}$

$\ln 2 = 0.5k \cdot \cancel{\ln e}$

$k = \frac{\ln 2}{0.5} = 1.386$

$y = 1e^{1.386(24)} = \boxed{2.815 \times 10^{14} \text{ bacteria}}$

b) Bacteria are reproducing quickly. Even though the person is fighting off some of the bacteria, they are growing too fast for the person's immune system to keep up.

25. $y = y_0 e^{-0.18t}$

$90 = 100e^{-0.18t}$

$0.9 = e^{-0.18t}$

$\ln 0.9 = -0.18t \cdot \cancel{\ln e}$

$t = \frac{\ln 0.9}{-0.18} = \boxed{0.585 \text{ day}}$

27. $(0,2) \rightarrow y_0 = 2$

$(2,5) \rightarrow t = 2, y = 5$

$y = y_0 e^{kt}$

$5 = 2e^{k(2)}$

$2.5 = e^{2k}$

$\ln 2.5 = 2k \cdot \cancel{\ln e}$

$k = \frac{\ln 2.5}{2} = 0.458$

$\boxed{y = 2e^{0.458t}}$

29. $y = y_0 e^{-kt}$

$t = \frac{3}{k}$, 95% has disintegrated or 5% remains

$y = 100e^{-k(3/k)} = 100e^{-3} = \frac{100}{e^3} = 4.979$

When $t = \frac{3}{k}$, 4.979% of the sample remains, so $\boxed{95.021\%}$ has decayed.