Section 7.4: 31-45 odd, 46, 54, 55, 57

31. $T_0 = 90,\ T = 60,\ t = 10,\ T_s = 20$

a) $T - T_s = (T_0 - T_s)e^{-kt}$

$60 - 20 = (90 - 20)e^{-k(10)}$

$40 = 70e^{-10k}$

$4/7 = e^{-10k}$

$\ln(4/7) = -10k \cdot \cancel{\ln e}$

$k = \dfrac{\ln(4/7)}{-10} = 0.0560$

Now $T = 35$

$35 - 20 = (90 - 20)e^{-0.0560t}$

$15 = 70e^{-0.0560t}$

$3/14 = e^{-0.0560t}$

$\ln(3/14) = -0.0560t \cdot \cancel{\ln e}$

$t = \dfrac{\ln(3/14)}{-0.0560} = 27.527$ min total

27.527 min total − 10 min already = $\boxed{17.527 \text{ min more}}$

b) $T_0 = 90,\ T = 35,\ t = ?,\ T_s = -15,\ k = -0.0560$ from above

$T - T_s = (T_0 - T_s)e^{kt}$

$35 - -15 = (90 - -15)e^{-0.0560t}$

$50 = 105e^{-0.0560t}$

$10/21 = e^{-0.0560t}$

$\ln(10/21) = -0.0560t \cdot \cancel{\ln e}$

$t = \dfrac{\ln(10/21)}{-0.0560} = \boxed{13.258 \text{ min}}$

33. a) $\boxed{T - T_s = 79.466 \cdot 0.932^t}$

b) $T = 79.466 \cdot 0.932^t + T_s$

$\boxed{T = 79.466 \cdot 0.932^t + 10}$

c) $12 = 79.466 \cdot 0.932^t + 10$

$2 = 79.466 \cdot 0.932^t$

$0.0252 = 0.932^t$

$\ln 0.0252 = t \cdot \ln 0.932$

$t = \dfrac{\ln 0.0252}{\ln 0.932} = \boxed{52.287 \text{s}}$

d) $T = 79.466 \cdot 0.932^0 + 10 = 79.466 + 10 = \boxed{89.466^\circ C}$

35. carbon-14 half-life = 5700 yr

$$1 = 2e^{k \cdot 5700}$$
$$0.5 = e^{5700k}$$
$$\ln 0.5 = 5700k \cdot \cancel{\ln e}$$
$$k = \frac{\ln 0.5}{5700} = -0.0001216$$

$$44.5 = 100e^{-0.0001216t}$$
$$0.445 = e^{-0.0001216t}$$
$$\ln 0.445 = -0.0001216t \cdot \cancel{\ln e}$$
$$t = \frac{\ln 0.445}{-0.0001216} = \boxed{6{,}658.300\text{ yr}}$$

37. $$1/3 = 1e^{k(5)}$$
$$1/3 = e^{5k}$$
$$\ln(1/3) = 5k \cdot \cancel{\ln e}$$
$$k = \frac{\ln(1/3)}{5} = -0.220$$

$$1 = 2e^{-0.220t}$$
$$0.5 = e^{-0.220t}$$
$$\ln 0.5 = -0.220t \cdot \cancel{\ln e}$$
$$t = \frac{\ln 0.5}{-0.220} = \boxed{3.155\text{ yr}}$$

39. $$800 = 1000e^{k(10)}$$
$$0.8 = e^{10k}$$
$$\ln 0.8 = 10k \cdot \cancel{\ln e}$$
$$k = \frac{\ln 0.8}{10} = -0.0223$$

Another 14 hr = 24 hr total

$$y = 1000e^{-0.0223(24)} = \boxed{585.350\text{kg}}$$

41. $\frac{dp}{dt} = kp$, so $p = p_0 e^{kh}$

a) $p_0 = 1013$, $p = 90$ when $h = 20$
$$90 = 1013e^{k(20)}$$
$$90/1013 = e^{20k}$$
$$\ln(90/1013) = 20k \cdot \cancel{\ln e}$$
$$k = \frac{\ln(90/1013)}{20} = -0.121$$
$$\boxed{p = 1013e^{-0.121h}}$$

b) $p = ?$ at $h = 50$
$$p = 1013e^{-0.121(50)} = \boxed{2.383\text{ millibars}}$$

c) $h = ?$ when $p = 900$
$$900 = 1013e^{-0.121h}$$
$$0.888 = e^{-0.121h}$$
$$\ln 0.888 = -0.121h \cdot \cancel{\ln e}$$
$$h = \frac{\ln 0.888}{-0.121} = \boxed{0.977\text{ km}}$$

43. $\frac{dV}{dt} = -\frac{1}{40}V$

a) $\int \frac{1}{V}\,dV = \int -\frac{1}{40}\,dt$

$\ln|V| = -\frac{1}{40}t + C$

$\cancel{e}^{\cancel{\ln}|V|} = e^{-1/40t + C}$

$|V| = e^{-1/40t} \cdot e^{C}$ V_0

$\boxed{V = V_0 e^{-1/40t}}$

b) $10 = 100e^{-1/40t}$

$0.1 = e^{-1/40t}$

$\ln 0.1 = -\frac{1}{40}t \cdot \cancel{\ln e}$

$t = \frac{\ln 0.1}{(-1/40)} = \boxed{92.103\text{ s}}$

45. $A = A_0 e^{rt}$

a) $90 = 1e^{r(100)}$

$90 = e^{100r}$

$\ln 90 = 100r \cdot \cancel{\ln e}$

$r = \frac{\ln 90}{100} = 0.04500 = \boxed{4.500\%}$

b) $131 = 1e^{r(100)}$

$131 = e^{100r}$

$\ln 131 = 100r \cdot \cancel{\ln e}$

$r = \frac{\ln 131}{100} = 0.04875 = \boxed{4.875\%}$

46. a) $A = A_0 e^{rt}$

$2 = 1e^{rt}$

$2 = e^{rt}$

$\ln 2 = rt \cdot \cancel{\ln e}$

$\boxed{t = \frac{\ln 2}{r}}$

b) Graph on calculator

c) $\ln 2 \approx 0.693$, which is close to 0.70 or 0.72.

d) $\frac{70}{i} = \frac{70}{5} = \boxed{14\text{ yr}}$

$\frac{72}{i} = \frac{72}{5} = \boxed{14.4\text{ yr}}$

e) Triple: $3 = 1e^{rt} \rightarrow 3 = e^{rt} \rightarrow \ln 3 = rt \cdot \cancel{\ln e} \rightarrow t = \frac{\ln 3}{r}$

$\ln 3 \approx 1.0986$, so estimate with $\boxed{\frac{108}{i} \text{ or } \frac{110}{i}}$

54. $v = v_0 e^{-(k/m)t}$

a) $s(t) = \int v_0 e^{-(k/m)t} = v_0 \int e^{-(k/m)t} = v_0 \cdot -\frac{m}{k} e^{-(k/m)t} + C$

$s(0) = 0$

$-\frac{v_0 m}{k}\cancel{e^{0}} + C = 0 \rightarrow C = \frac{v_0 m}{k}$

$s(t) = -\frac{v_0 m}{k} e^{-(k/m)t} + \frac{v_0 m}{k} = \frac{v_0 m}{k}\left(-e^{-(k/m)t} + 1\right) = \boxed{\frac{v_0 m}{k}\left(1 - e^{-(k/m)t}\right)}$

b) $\lim_{t\to\infty} s(t) = \lim_{t\to\infty} \frac{v_0 m}{k}\left(1 - \frac{1}{e^{(k/m)\infty}}\right) = \frac{v_0 m}{k}\left(1 - \frac{1}{\infty}\right) = \frac{v_0 m}{k}(1 - 0) = \boxed{\frac{v_0 m}{k}}$

55. $V_0 = 0.80$ m/s, $m = 49.90$ kg, $d = \frac{V_0 m}{k} = 1.32$ m

$s(t) = \frac{V_0 m}{k}\left(1 - e^{-(k/m)t}\right)$ from question 54

$k = \frac{V_0 m}{d} = \frac{0.80(49.90)}{1.32} = 30.242$

$s(t) = 1.32\left(1 - e^{-(30.242/49.90)t}\right) = \boxed{1.32\left(1 - e^{-0.606t}\right)}$

Graph on calculator

57. $\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x = e$

a)

x	$\left(1+\frac{1}{x}\right)^x$
10	2.594
100	2.705
1,000	2.717
10,000	2.7181
100,000	2.7183
1,000,000	2.7183

$e \approx 2.7183$ ✓

b) $r = 2$

$\lim_{x\to\infty}\left(1+\frac{2}{x}\right)^x = e^2$

x	$\left(1+\frac{2}{x}\right)^x$
10	6.192
100	7.245
1,000	7.374
10,000	7.388
100,000	7.3889
1,000,000	7.3890

$e^2 \approx 7.3890$ ✓

b) $r = 0.5$

$\lim_{x\to\infty}\left(1+\frac{0.5}{x}\right)^x = e^{0.5}$

x	$\left(1+\frac{0.5}{x}\right)^x$
10	1.629
100	1.647
1,000	1.6485
10,000	1.6487
100,000	1.64872
1,000,000	1.648721

$e^{0.5} \approx 1.648721$ ✓

c) When interest is compounded over smaller periods of time, interest is being given more frequently. Interest is given increasingly often. As the number of compounding periods approaches infinity, the interest approaches being compounded continuously.