Section 7.4 (Day 2)

* Newton's Law of Cooling/Warming:

$$T - T_s = (T_0 - T_s)\, e^{-kt}$$

- T: Temp of the Object
- T_s: Temp of Surroundings
- T_0: Initial Temp
- t: time

#31

ex: $T_0 = 90°$, $T = 60°$, $t = 10$ mins, $T_s = 20°$

$60 - 20 = (90-20)e^{-k(10)}$

$\frac{40}{70} = \frac{70e^{-10k}}{70}$

$\frac{\ln(4/7)}{-10} = \frac{-10k}{-10}$

$k = 0.05596\ldots$ (ANS)

a) $T = 35°$, $T_0 = 60°$, $t = ?$, $T_s = 20°$

$35 - 20 = (60-20)e^{-ANS(t}$

$15 = 40e^{-ANS \cdot t}$

$\frac{\ln(15/40)}{-ANS} = \frac{-ANS \cdot t}{-ANS}$

$t = 17.527$ mins

b) $T_0 = 90°$, $T_s = 15°$, $T = 35°$, $t = ?$

$35 - 15 = (90-15)e^{-ANS \cdot t}$

$50 = 105e^{-ANS \cdot t}$

$\frac{\ln(50/105)}{-ANS} = \frac{-ANS \cdot t}{-ANS}$

$t = 13.258$ mins

* Resistance Proportional to Velocity:

$$V = V_0 e^{-(k/m)t}$$

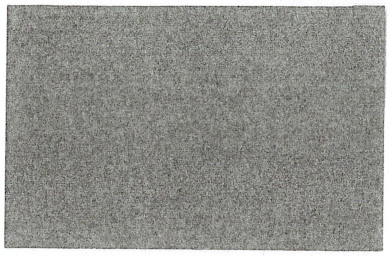
- V_0: initial velocity
- m: mass
- t: time

ex: A 50-kg iceskater has a velocity constant of $k = 2.1$ kg/sec. How long will the skater coast from 7 m/sec to 1 m/sec?

$1 = 7e^{-(2.1/50)t}$

$\frac{\ln(1/7)}{-2.1/50} = \frac{+2.1/50 \cdot t}{+2.1/50}$

$t = 46.331$ sec



Let $v(t)$ be the velocity, in feet per second, of a skydiver at time t seconds, $t \geq 0$. After her parachute opens, her velocity satisfies the differential equation $\frac{dv}{dt} = -2v - 32$, with initial condition $v(0) = -50$.

(a) Use separation of variables to find an expression for v in terms of t, where t is measured in seconds.

(b) Terminal velocity is defined as $\lim_{t \to \infty} v(t)$. Find the terminal velocity of the skydiver to the nearest foot per second.

(c) It is safe to land when her speed is 20 feet per second. At what time t does she reach this speed?

+5 a) $\frac{dv}{dt} = -2v - 32 = -2(v+16)$

$\int \frac{1}{v+16}\, dv = \int -2\, dt$

$\ln|v+16| = -2t + C$

$e^{\ln|v+16|} = e^{-2t+C}$

$v+16 = e^{-2t+C} = e^{-2t} \cdot e^{C} = A$

$v+16 = Ae^{-2t} \longrightarrow v+16 = -34e^{-2t}$

$v(0) = -50 \rightarrow -50+16 = Ae^{0}$

$-34 = A$

$\boxed{v = -34e^{-2t} - 16}$

+2 b) $\lim_{t \to \infty} (-34e^{-2t} - 16) = \lim_{t \to \infty} \left(\frac{-34}{e^{2t}} - 16\right) = \boxed{-16 \text{ ft/sec}}$

+2 c) $-20 = -34e^{-2t} - 16$

$+16 \quad +16$

$-4 = -34e^{-2t}$

$\frac{\ln(4/34)}{-2} = \frac{-2t}{-2}$

$\boxed{t = 1.070 \text{ sec}}$