# Section 7.5 : 1-37 odd

1. $\dfrac{x-12}{x^2-4x} = \dfrac{A}{x} + \dfrac{B}{x-4}$

$A(x-4) + B(x) = x-12$

$x=4: \quad 4B = -8 \rightarrow \boxed{B=-2}$

$x=0: \quad -4A = -12 \rightarrow \boxed{A=3}$

3. $\dfrac{16-x}{x^2+3x-10} = \dfrac{A}{x-2} + \dfrac{B}{x+5}$

$A(x+5) + B(x-2) = 16-x$

$x=-5: \quad -7B = 21 \rightarrow \boxed{B=-3}$

$x=2: \quad 7A = 14 \rightarrow \boxed{A=2}$

5. $\displaystyle\int \frac{x-12}{x^2-4x} = \int\left(\frac{3}{x}+\frac{-2}{x-4}\right)dx = 3\ln|x| - 2\ln|x-4| + C$

(from question 1)

$= \ln|x|^3 + \ln|x-4|^{-2} + C$

$= \boxed{\ln\dfrac{|x|^3}{(x-4)^2} + C}$

7. $\displaystyle\int \frac{2x^3}{x^2-4} \rightarrow$ long division bc degree of numerator > degree of denominator

$$\begin{array}{r} 2x \\ x^2-4\,\big)\,\cancel{2x^3} \\ -(\cancel{2x^3}-8x) \\ \hline 8x \end{array}$$

$\displaystyle\int\left(2x + \frac{8x}{x^2-4}\right)dx$

$u = x^2-4$

$du = 2x\,dx$

$dx = \dfrac{du}{2x}$

$\displaystyle\int 2x\,dx + \int\frac{8x}{x^2-4}dx = x^2 + \int\frac{8\cancel{x}}{u}\cdot\frac{du}{2\cancel{x}} = x^2 + \int 4\cdot\frac{1}{u}du = x^2 + 4\ln|u| + C$

$= x^2 + 4\ln|x^2-4| + C$

$= \boxed{x^2 + \ln(x^2-4)^4 + C}$

9. $\displaystyle\int\frac{2}{x^2+1}dx = \int 2\cdot\frac{1}{x^2+1}dx = \boxed{2\tan^{-1}x + C}$

11. $\int \frac{7}{2x^2-5x-3}\,dx$

$2x^2-6x+x-3$

$2x(x-3)+1(x-3) = (x-3)(2x+1)$

$$\frac{7}{2x^2-5x-3} = \frac{A}{x-3} + \frac{B}{2x+1}$$

$A(2x+1) + B(x-3) = 7$

$x=-1/2$ : $-3.5B = 7 \rightarrow B = -2$

$x = 3$ : $7A = 7 \rightarrow A = 1$

$$\int\left(\frac{1}{x-3} + \frac{-2}{2x+1}\right)dx = \ln|x-3| - \ln|2x+1| + C = \boxed{\ln\left|\frac{x-3}{2x+1}\right| + C}$$

13. $\int \frac{8x-7}{2x^2-x-3}$

$2x^2-3x+2x-3$

$x(2x-3)+1(2x-3) = (2x-3)(x+1)$

$$\frac{8x-7}{2x^2-x-3} = \frac{A}{2x-3} + \frac{B}{x+1}$$

$A(x+1) + B(2x-3) = 8x-7$

$x=-1$ : $-5B = -15 \rightarrow B = 3$

$x = 3/2$ : $2.5A = 5 \rightarrow A = 2$

$$\int\left(\frac{2}{2x-3} + \frac{3}{x+1}\right)dx = \ln|2x-3| + 3\ln|x+1| + C = \boxed{\ln\left(|2x-3|\cdot|x+1|^3\right) + C}$$

15. $y = \int \frac{2x-6}{x^2-2x}\,dx$

$$\frac{2x-6}{x^2-2x} = \frac{A}{x} + \frac{B}{x-2}$$

$A(x-2) + B(x) = 2x-6$

$x = 2$ : $2B = -2 \rightarrow B = -1$

$x = 0$ : $-2A = -6 \rightarrow A = 3$

$$y = \int\left(\frac{3}{x} + \frac{-1}{x-2}\right)dx = 3\ln|x| - \ln|x-2| + C = \boxed{\ln\frac{|x|^3}{|x-2|} + C}$$

17. $F(x) = \int \frac{2}{x^3 - x}\,dx$

$$\frac{2}{x^3-x} = \frac{2}{x(x^2-1)} = \frac{2}{x(x+1)(x-1)} = \frac{A}{x} + \frac{B}{x+1} + \frac{C}{x-1}$$

$$A(x+1)(x-1) + B(x)(x-1) + C(x)(x+1) = 2$$

$x = 0: \quad -A = 2 \rightarrow \quad A = -2$

$x = -1: \quad 2B = 2 \rightarrow \quad B = 1$

$x = 1: \quad 2C = 2 \rightarrow \quad C = 1$

$$F(x) = \int \left( \frac{-2}{x} + \frac{1}{x+1} + \frac{1}{x-1} \right) dx = -2\ln|x| + \ln|x+1| + \ln|x-1| + C$$

$$= \ln|x|^{-2} + \ln|x+1| + \ln|x-1| + C$$

$$= \ln \frac{|x+1| \cdot |x-1|}{|x|^2} + C = \boxed{\ln \frac{|x^2-1|}{x^2} + C}$$

19. $\int \frac{2x}{x^2-4}\,dx$

$u = x^2 - 4$

$du = 2x\,dx$

$dx = \frac{du}{2x}$

$$\int \frac{\cancel{2x}}{u} \cdot \frac{du}{\cancel{2x}} = \int \frac{1}{u}\,du = \ln|u| + C = \boxed{\ln|x^2-4| + C}$$

21. $\int \frac{x^2 + x - 1}{x^2 - x}\,dx$

$$\begin{array}{r} 1 \phantom{xxxxxx} \\ x^2 - x \,\big)\, \overline{\cancel{x^2} + x - 1} \\ -(\cancel{x^2} - x) \phantom{xx} \\ \hline 2x - 1 \end{array}$$

$$\int \left( 1 + \frac{2x-1}{x^2-x} \right) dx = \boxed{x + \ln|x^2 - x| + C}$$

23. $\frac{dP}{dt} = \overset{k}{0.006}P(\overset{M}{200} - P) = kP(M-P)$

a) $M = \boxed{200}$

b) Growing fastest at $\frac{1}{2}M = \frac{1}{2}(200) = \boxed{100}$

c) $\frac{dP}{dt}(100) = 0.006 \cdot 100(200-100) = \boxed{60}$

25. $\frac{dP}{dt} = \overset{k}{0.0002}P(\overset{M}{1200} - P) = kP(M-P)$

a) $M = \boxed{1200}$

b) Growing fastest at $\frac{1}{2}M = \frac{1}{2}(1200) = \boxed{600}$

c) $\frac{dP}{dt}(600) = 0.0002 \cdot 600(1200-600) = \boxed{72}$

27. $\frac{dP}{dt} = \overset{k}{0.006}P(\overset{M}{200} - P) = kP(M-P)$ and $P=8$ when $t=0$

$$P = \frac{M}{1+Ae^{-(Mk)t}} \rightarrow 8 = \frac{200}{1+Ae^{-(200 \cdot 0.006)(0)}} \rightarrow 8 = \frac{200}{1+A}$$

$8(1+A) = 200 \rightarrow 1+A = 25 \rightarrow A = 24$

$$\boxed{P = \frac{200}{1+24e^{-1.2t}}}$$

29. $\frac{dP}{dt} = \overset{k}{0.0002}P(\overset{M}{1200} - P) = kP(M-P)$ and $P=20$ when $t=0$

$$P = \frac{M}{1+Ae^{-(Mk)t}} \rightarrow 20 = \frac{1200}{1+Ae^{-(1200 \cdot 0.0002)(0)}} \rightarrow 20 = \frac{1200}{1+A}$$

$20(1+A) = 1200 \rightarrow 1+A = 60 \rightarrow A = 59$

$$\boxed{P = \frac{1200}{1+59e^{-0.24t}}}$$

31. $P(t) = \dfrac{1000}{1+e^{4.8-0.7t}} = \dfrac{1000}{1+e^{4.8}\cdot e^{-0.7t}} = \dfrac{1000}{1+121.510e^{-0.7t}} = \dfrac{M}{1+Ae^{-(Mk)t}}$

a) $M = \boxed{1000}$ (numerator)

$Mk = 0.7 \rightarrow k = \dfrac{0.7}{M} = \dfrac{0.7}{1000} = \boxed{0.0007}$

b) $P(0) = \dfrac{1000}{1+e^{4.8}} = 8.162 \approx$ $\boxed{\text{8 rabbits initially}}$

33. $\dfrac{dP}{dt} = \overset{k}{0.0015}P(\overset{M}{150}-P) = kP(M-P)$ and $P=6$ when $t=0$

a) $P = \dfrac{M}{1+Ae^{-(Mk)t}} \rightarrow 6 = \dfrac{150}{1+Ae^{-(150\cdot 0.0015)(0)}} \rightarrow 6 = \dfrac{150}{1+A}$

$6(1+A) = 150 \rightarrow 1+A = 25 \rightarrow A = 24 \rightarrow$ $\boxed{P = \dfrac{150}{1+24e^{-0.225t}}}$

b) $100 = \dfrac{150}{1+24e^{-0.225t}} \rightarrow 1+24e^{-0.225t} = \dfrac{150}{100} \rightarrow 1+24e^{-0.225t} = 1.5$

$24e^{-0.225t} = 0.5 \rightarrow e^{-0.225t} = 1/48 \rightarrow -0.225t\cdot\cancel{\ln e} = \ln(1/48)$

$t = \ln(1/48)/-0.225 = \boxed{17.205 \text{ weeks}}$

$125 = \dfrac{150}{1+24e^{-0.225t}} \rightarrow 1+24e^{-0.225t} = \dfrac{150}{125} \rightarrow 1+24e^{-0.225t} = \dfrac{6}{5}$

$24e^{-0.225t} = 0.2 \rightarrow e^{-0.225t} = \dfrac{1}{120} \rightarrow -0.225t\cdot\cancel{\ln e} = \ln(1/120)$

$t = \ln(1/120)/-0.225 = \boxed{21.278 \text{ weeks}}$

35. $\dfrac{dP}{dt} = kP(M-P)$

$\dfrac{1}{P(M-P)}\,dP = k\,dt$

$\dfrac{1}{P(M-P)} = \dfrac{A}{P} + \dfrac{B}{M-P}$

$A(M-P) + B(P) = 1$

$P=0: \; A\cdot M = 1 \rightarrow A = 1/M$

$P=M: \; B\cdot M = 1 \rightarrow B = 1/M$

$$\int \frac{1}{P(M-P)}\,dP = \int\left(\frac{1/M}{P} + \frac{1/M}{M-P}\right)dP = \frac{1}{M}\int\left(\frac{1}{P}+\frac{1}{M-P}\right)dP$$

$\longrightarrow$

35. (continued)

$$\frac{1}{M}\int\left(\frac{1}{P}+\frac{1}{M-P}\right)dP = \int k\,dt$$

$$\frac{1}{M}\int\left(\frac{1}{P}+\frac{1}{M-P}\right)dP = kt + C$$

$$\int\left(\frac{1}{P}+\frac{1}{M-P}\right)dP = Mkt + C$$

Multiply by $-1$ on both sides. Note: $\frac{-1}{M-P} = \frac{1}{P-M}$

$$\int\left(\frac{1}{P-M}-\frac{1}{P}\right)dP = -Mkt + C$$

$$\ln|P-M| - \ln|P| = -Mkt + C$$

$$\ln\left|\frac{P-M}{P}\right| = -Mkt + C$$

$$\cancel{\ln}\left|1-\frac{M}{P}\right| = -Mkt + C$$

$e \qquad\qquad e$

$$1-\frac{M}{P} = e^{-Mkt}\cdot e^{C}$$ Call this constant A

$$1-\frac{M}{P} = Ae^{-Mkt}$$

$$\frac{M}{P} = 1 + Ae^{-Mkt}$$ (A could be + or −)

$$\frac{P}{M} = \frac{1}{1+Ae^{-Mkt}}$$

$$\boxed{P = \frac{M}{1+Ae^{-Mkt}}}$$

37. a) $y = \dfrac{232{,}739.875}{1+14.582e^{-0.101x}}$

b) 232,740 people

c) 59.896 → 60 years after 1950 → 2010

d) $M = 232{,}740$

$M\cdot k = 0.101 \rightarrow k = \dfrac{0.101}{M} = \dfrac{0.101}{232{,}740} = 4.340\times10^{-7}$

$$\frac{dP}{dt} = kP(M-P) = 4.340\times10^{-7}P(232{,}740-P)$$