Integral as Net Change (Section 8.1)

* Linear Motion: when $v>0$, the particle moves RIGHT
  $v<0$, the particle moves LEFT

* Displacement = (rate of change) * (time) OR $\int v(t)\,dt$

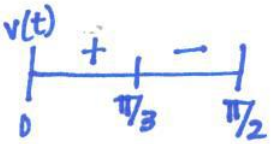
** New position = Initial position + displacement

* Total distance = |displacement| OR $\int |v(t)|\,dt$

ex: $v(t) = 6\sin 3t \quad 0 \le t \le \pi/2$

a) left, right, stopped? ($v(t)<0$, $v(t)>0$, $v(t)=0$) → $6\sin 3t = 0$

$t = 0, \pi/3, 2\pi/3, \ldots$



left: $(\pi/3, \pi/2]$

right: $(0, \pi/3)$

stopped: $t = 0, \pi/3$

b) displacement? $\int v(t)\,dt$

c) total distance? $\int |v(t)|\,dt$

$$\int_0^{\pi/3} 6\sin 3t\,dt - \int_{\pi/3}^{\pi/2} 6\sin 3t\,dt$$

$$-2\cos 3t \Big|_0^{\pi/3} + 2\cos 3t \Big|_{\pi/3}^{\pi/2}$$

$$-2\left(\cos\pi - \cos 0\right) + 2\left(\cos\tfrac{3\pi}{2} - \cos\pi\right)$$

$$= -2(-2) + 2(1)$$

$$= 6$$

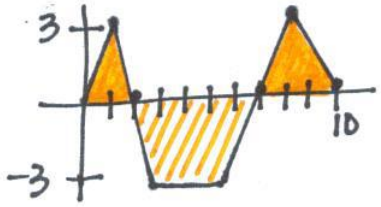
$$\int_0^{\pi/2} 6\sin 3t\,dt$$

$$\frac{-6\cos 3t}{3}\Big|_0^{\pi/2}$$

$$-2\left[\cos\tfrac{3\pi}{2} - \cos 0\right] = -2(-1)$$

$$= 2$$

#20



when $t=0$, $x=2$

a) Final position? → s_0 + displacement $= 2 + \int_0^{10} v(t)\,dt$

b) Distance Traveled? $\int_0^{10} |v(t)|\,dt$

$= 3 + 12 + 4.5$

$= 19.5$ meters

$= 2 + \left[\underbrace{\tfrac{1}{2}(3)(2)}_{\text{Triangle}} + \underbrace{\tfrac{1}{2}(3+5)(-3)}_{\text{Trapezoid}} + \underbrace{\tfrac{1}{2}(3)(3)}_{\text{Triangle}}\right]$

$= 2 + [3 - 12 + 4.5]$

$= -2.5$

* pg 386 Example #4 (helps w/ #11 on HW)

$V_0 = 5$ $\quad a(t) = 2.4t$ mph per second for 8 sec.

a) How fast is the car going when 8 sec are up? $V_0 + \int_0^8 a(t)\,dt$

b) How far did it travel during those 8 sec? $\int_0^8 |v(t)\,dt|$

$= 5 + \int_0^8 2.4t\,dt$

$= 5 + 1.2t^2 \Big|_0^8$

$= 5 + 1.2(8^2 - 0^2)$

$= 81.8$ mph

$= \int_0^8 |5 + 1.2t^2|\,dt$

$= 244.8$ mph $\times$ sec

$= 244.8 \frac{\text{miles}}{\text{hr}} \times \frac{1\,\text{hr}}{3600\,\text{sec}} \times$ sec

$= 0.068$ miles

#5 $v(t) = 5\sin^2 t \cos t \quad 0 \le t \le 2\pi \quad s(0) = 3$

a) left, right, stopped $\longrightarrow$ $0 = 5\sin^2 t \cos t$ $\longrightarrow$ $t = 0, \pi, 2\pi \quad t = \pi/2, 3\pi/2$

b) Displacement $\neq$ final position

c) Total distance

$v(t)$: + | − | − | + over $0, \pi/2, \pi, 3\pi/2, 2\pi$

left: $(\pi/2, \pi) \cup (\pi, 3\pi/2)$
right: $(0, \pi/2) \cup (3\pi/2, 2\pi)$
stopped: $t = 0, \pi/2, \pi, 3\pi/2, 2\pi$

F.P. $= s_0 + \int_0^{2\pi} 5\sin^2 t \cos t \, dt$ (displacement)

Displacement: 0

$u = \sin t$
$du = \cos t \, dt$
$u(0) = \sin 0 = 0$
$u(2\pi) = \sin 2\pi = 0$

F.P. $= 3 + \int_0^0 5u^2 \, du$

F.P. $= 3$

$= \int_0^{2\pi} |5\sin^2 t \cos t| \, dt$ (in calc)

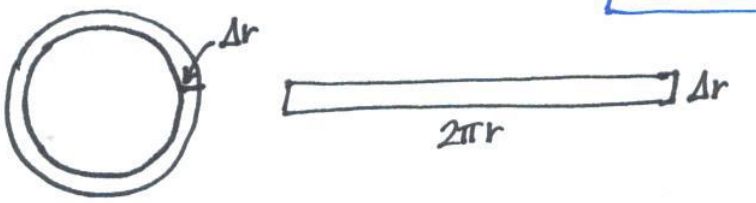
$= 6.\overline{6}$ or $\frac{20}{3}$

#23 Pop Density $= 10{,}000(2-r)$

a) If Pop density $\to 0$, find r. $0 = 10{,}000(2-r)$

$r = 2$ miles

b)



$A = (2\pi r)\Delta r$

c) Population $= \underbrace{10{,}000(2-r)}_{\text{Pop density}} \underbrace{(2\pi r)\Delta r}_{\text{Area}}$ why?

d) $\int_0^2 10{,}000(2-r)(2\pi r) \, dr$ (in calc)

$= 83{,}775.8041$

$\approx 83{,}775$