Section 8.1: 1-27 odd

1. $v(t) = 5\cos t, \quad 0 \le t \le 2\pi$

a) $v(t) = 0 \rightarrow 5\cos t = 0 \rightarrow \cos t = 0$ at $\boxed{t = \pi/2,\ 3\pi/2}$ (stopped)

$v(t)$: + + 0 − − − − 0 + + on $[0, 2\pi]$, with zeros at $\pi/2$ and $3\pi/2$ ($5\cos t$)

Right: $v(t) > 0$, so $\boxed{(0, \pi/2) \cup (3\pi/2, 2\pi)}$

Left: $v(t) < 0$, so $\boxed{(\pi/2, 3\pi/2)}$

b) $$\int_0^{2\pi} 5\cos t\,dt = 5\sin t\Big|_0^{2\pi} = 5\sin 2\pi - 5\sin 0 = 5(0) - 5(0) = 0 - 0 = \boxed{0}$$

$3 + 0 = \boxed{3}$

c) $$\int_0^{\pi/2} 5\cos t\,dt = 5\sin t\Big|_0^{\pi/2} = 5\sin\tfrac{\pi}{2} - 5\sin 0 = 5(1) - 5(0) = 5 - 0 = 5$$

$$\int_{\pi/2}^{3\pi/2} 5\cos t\,dt = 5\sin t\Big|_{\pi/2}^{3\pi/2} = 5\sin\tfrac{3\pi}{2} - 5\sin\tfrac{\pi}{2} = 5(-1) - 5(1) = -5 - 5 = -10$$

$$\int_{3\pi/2}^{2\pi} 5\cos t\,dt = 5\sin t\Big|_{3\pi/2}^{2\pi} = 5\sin 2\pi - 5\sin\tfrac{3\pi}{2} = 5(0) - 5(-1) = 0 + 5 = 5$$

Total distance traveled $= 5 + |-10| + 5 = 5 + 10 + 5 = \boxed{20}$

3. $v(t) = 49 - 9.8t, \quad 0 \le t \le 10$

a) $v(t) = 0 \rightarrow 49 - 9.8t = 0 \rightarrow 9.8t = 49 \rightarrow \boxed{t = 5}$ (stopped)

$v(t)$: + + + 0 − − − on $[0, 10]$, with zero at 5 ($49 - 9.8t$)

Right: $\boxed{(0, 5)}$

Left: $\boxed{(5, 10)}$

b) $$\int_0^{10} (49 - 9.8t)\,dt = 49t - 4.9t^2\Big|_0^{10} = 49(10) - 4.9(10)^2 = 490 - 490 = \boxed{0}$$

$3 + 0 = \boxed{3}$

c) $$\int_0^{5} (49 - 9.8t)\,dt = 49t - 4.9t^2\Big|_0^{5} = 49(5) - 4.9(5)^2 = 122.5$$

3. c) $\displaystyle\int_5^{10} (49-9.8t)\,dt = 49t - 4.9t^2 \Big|_5^{10} = (49\cdot 10 - 4.9\cdot 10^2) - (49\cdot 5 - 4.9\cdot 5^2) = -122.5$

Total distance traveled $= 122.5 + |-122.5| = \boxed{245}$

5. $v(t) = 5\sin^2 t\cos t,\ 0 \le t \le 2\pi$

a) $v(t) = 0 \rightarrow 5\sin^2 t\cos t = 0$ when $\sin t = 0$ or $\cos t = 0$

Stopped when $\boxed{t = 0, \pi/2, \pi, 3\pi/2, 2\pi}$

$\sin^2 t$ is always +, so the sign of $v(t)$ depends on the sign of $\cos t$.

$v(t)$: 0 + + 0 − − 0 − − 0 + + 0 at $0,\ \pi/2,\ \pi,\ \frac{3\pi}{2},\ 2\pi$

Right: $\boxed{(0, \pi/2) \cup (3\pi/2, 2\pi)}$

Left: $\boxed{(\pi/2, \pi) \cup (\pi, 3\pi/2)}$

b) $\displaystyle\int_0^{2\pi} 5\sin^2 t\cos t\,dt = \frac{5}{3}(\sin t)^3 \Big|_0^{2\pi} = \frac{5}{3}(\sin 2\pi)^3 - \frac{5}{3}(\sin 0)^3 = \frac{5}{3}(0) - \frac{5}{3}(0) = \boxed{0}$

$3 + 0 = \boxed{3}$

c) $\displaystyle\int_0^{\pi/2} 5\sin^2 t\cos t\,dt = \frac{5}{3}(\sin t)^3 \Big|_0^{\pi/2} = \frac{5}{3}(1^3 - 0^3) = \frac{5}{3}\cdot 1 = \frac{5}{3}$

$\displaystyle\int_{\pi/2}^{\pi} 5\sin^2 t\cos t\,dt = \frac{5}{3}(\sin t)^3 \Big|_{\pi/2}^{\pi} = \frac{5}{3}(0^3 - 1^3) = \frac{5}{3}\cdot(-1) = \frac{-5}{3}$

$\displaystyle\int_{\pi}^{3\pi/2} 5\sin^2 t\cos t\,dt = \frac{5}{3}(\sin t)^3 \Big|_{\pi}^{3\pi/2} = \frac{5}{3}((-1)^3 - 0^3) = \frac{5}{3}(-1) = \frac{-5}{3}$

$\displaystyle\int_{3\pi/2}^{2\pi} 5\sin^2 t\cos t\,dt = \frac{5}{3}(\sin t)^3 \Big|_{3\pi/2}^{2\pi} = \frac{5}{3}(0^3 - (-1)^3) = \frac{5}{3}\cdot 1 = \frac{5}{3}$

Total distance traveled $= \frac{5}{3} + \left|\frac{-5}{3}\right| + \left|\frac{-5}{3}\right| + \frac{5}{3} = \boxed{\frac{20}{3}}$

7. $v(t) = e^{\sin t} \cos t$, $0 \le t \le 2\pi$

a) $e^{\sin t} \cdot \cos t = 0$ when $\cos t = 0 \to \boxed{t = \pi/2, 3\pi/2}$ (stopped)

$v(t)$ $\underset{0 \quad\quad \frac{\pi}{2} \quad\quad \frac{3\pi}{2} \quad\quad 2\pi}{[\,+\,+\,0\,-\,-\,0\,+\,+\,]}$ $\quad \underbrace{e^{\sin t}}_{+} \cdot \cos t$ Right : $\boxed{(0, \pi/2) \cup (3\pi/2, 2)}$

Left : $\boxed{(\pi/2, 3\pi/2)}$

b) $\int_0^{2\pi} e^{\sin t} \cdot \cos t \, dt = e^{\sin t}\Big|_0^{2\pi} = e^{\sin 2\pi} - e^{\sin 0} = e^0 - e^0 = 1 - 1 = \boxed{0}$

$3 + 0 = \boxed{3}$

c) $\int_0^{\pi/2} e^{\sin t} \cos t \, dt = e^{\sin t}\Big|_0^{\pi/2} = e^1 - e^0 = e - 1$

$\int_{\pi/2}^{3\pi/2} e^{\sin t} \cos t \, dt = e^{\sin t}\Big|_{\pi/2}^{3\pi/2} = e^{-1} - e^1 = \frac{1}{e} - e$

$\int_{3\pi/2}^{2\pi} e^{\sin t} \cos t \, dt = e^{\sin t}\Big|_{3\pi/2}^{2\pi} = e^0 - e^{-1} = 1 - \frac{1}{e}$

Total distance $= e - 1 + \left|\frac{1}{e} - e\right| + 1 - \frac{1}{e} = e - \cancel{1} + e - \frac{1}{e} + \cancel{1} - \frac{1}{e} = \boxed{2e - \frac{2}{e}}$

$a(t) = 1 + 3\sqrt{t}$

9. $\int_0^9 (1 + 3\sqrt{t})\,dt = \int_0^9 (1 + 3t^{1/2})\,dt = t + 2t^{3/2}\Big|_0^9 = 9 + 2 \cdot 9^{3/2} = 9 + 2 \cdot 27 = \boxed{63 \text{ mph}}$

$v(t) = t + 2t^{3/2}$

$\int_0^9 (t + 2t^{3/2})\,dt = \frac{1}{2}t^2 + \frac{4}{5}t^{5/2}\Big|_0^9 = \frac{1}{2} \cdot 9^2 + \frac{4}{5} \cdot 9^{5/2} = \frac{234.9 \text{ mi}}{\text{hr}} \cdot s$

$\frac{234 \cancel{\text{mi}}}{\cancel{\text{hr}}} \cdot \cancel{s} \cdot \frac{1 \cancel{\text{hr}}}{60 \cancel{\text{min}}} \cdot \frac{1 \cancel{\text{min}}}{60 \cancel{s}} \cdot \frac{5{,}280 \text{ ft}}{1 \cancel{\text{mi}}} = \boxed{344.52 \text{ ft}}$

11. $a = -32 \text{ ft/s}^2$

a) $\int_0^3 -32\,dt = -32t \Big|_0^3 = -32(3) + 32(0) = -96 \text{ ft/s}$

$90 \text{ ft/s} - 96 \text{ ft/s} = \boxed{-6 \text{ ft/s}}$

b) $v(t) = -32t + 90$

$s(t) = -16t^2 + 90t + C \rightarrow 0 = 0 + 0 + C \rightarrow C = 0$

$-16t^2 + 90t = 0 \rightarrow 2t(-8t + 45) = 0 \rightarrow t = 0 \text{ or } 45 = 8t \rightarrow \boxed{t = 5.625 \text{ s}}$

c) $\boxed{0}$ (no net change from initial position)

d) $v(t) = 0 \rightarrow -32t + 90 = 0 \rightarrow t = 2.8125$

$v(t)$: + + + at 0 to 2.8125, − − − from 2.8125 to 5.625

$\int_0^{2.8125} (-32t + 90)\,dt = -16t^2 + 90t \Big|_0^{2.8125} = 126.5625$

$\int_{2.8125}^{5.625} (-32t + 90)\,dt = -16t^2 + 90t \Big|_{2.8125}^{5.625} = -126.5625$

Total distance traveled $= 126.5625 + |-126.5625| = \boxed{253.125 \text{ ft}}$

13. $4 + 5 + |-24| = 4 + 5 + 24 = \boxed{33 \text{ cm}}$

15. Greatest acceleration = greatest slope of velocity graph: $\boxed{t = a}$

17. 4 Δ's

$4 \cdot \frac{1}{2} \cdot 1 \cdot 2 = 4$ units right

a) $2 + 4 = \boxed{6}$

b) $\boxed{4}$

19. $-\frac{1}{2}(1)(2) = -1$ (left)

$\frac{1}{2} \cdot 2(1+4) = 1.5 = 5$ (right)

$-\frac{1}{2} \cdot 2 \cdot 1 = -1$ (left)

a) $2 - 1 + 5 - 1 = \boxed{5}$

b) $1 + 5 + 1 = \boxed{7}$

21. $\displaystyle\int_0^{10} 27.08 e^{\frac{1}{25}t}\,dt = 25\cdot 27.08 e^{\frac{1}{25}t}\Big|_0^{10} = 25\cdot 27.08\left(e^{10/25} - e^{0}\right) = \boxed{332.965 \text{ billion barrels}}$

23. Density $= 10{,}000(2-r)$

a) $10{,}000(2-r)$ approaches 0 as r approaches 2, so $\boxed{r = 2\text{ mi}}$

b) $A = lw = Cw = \boxed{2\pi r\,\Delta r}$

c) Population size $=$ Population Density $\times$ Area

$= 10{,}000(2-r)\cdot 2\pi r\,\Delta r = 20{,}000\pi r(2-r)\,\Delta r$

d) $\displaystyle\int_0^{2} 20{,}000\pi(2r - r^2)\,dr = 20{,}000\pi\left(r^2 - \tfrac{1}{3}r^3\right)\Big|_0^{2} = 83{,}775.804 \rightarrow \boxed{83{,}776 \text{ people}}$

25. a) $5 + 8.9 + 16 + \ldots + 155.3 + 188 = \boxed{797.5 \text{ thousand bagels}}$

b) $\boxed{B(x) = 1.6x^2 + 2.3x + 5}$

c) $\displaystyle\int_0^{11} (1.6x^2 + 2.3x + 5)\,dx = \frac{1.6}{3}x^3 + \frac{2.3}{2}x^2 + 5x\Big|_0^{11} = \boxed{904.017 \text{ thousand bagels}}$

d) The answer to part a represents left hand rectangles. The answer to part c represents the exact area under the curve, which is greater than LRAM since the curve is increasing on $[0, 11]$.

27. $T = \frac{1}{2}\cdot 1\left(120 + 2(110) + 2(115) + 2(115) + \ldots + 2(110) + 121\right) = \boxed{1{,}156.5 \text{ cases of milk}}$