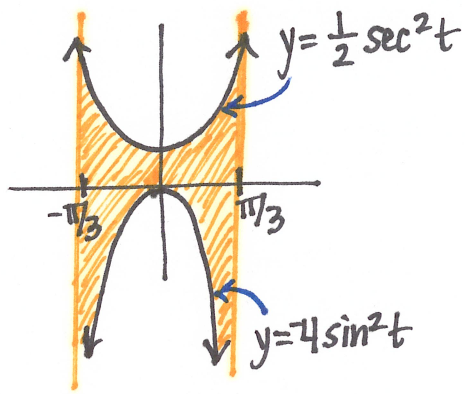
Areas in the Plane (Section 8.2)

* To find area between two curves: $A = \int_a^b [f(x) - g(x)]\,dx$

  * upper graph − lower graph when using "dx" → equations are "y = __"
    right graph − left graph when using "dy" → equations are "x = __"

ex:



$-\pi/3 \le t \le \pi/3$

$$\int_{-\pi/3}^{\pi/3} \left[\left(\tfrac{1}{2}\sec^2 t\right) + \left(+4\sin^2 t\right)\right] dt \quad \underline{\underline{\text{OR}}}$$

use symmetry and get:

$$= 2\int_0^{\pi/3} \left[\left(\tfrac{1}{2}\sec^2 t\right) + \left(+4\sin^2 t\right)\right] dt$$

pg. 620 Integral Table #58

$$= \int_0^{\pi/3} \left(\sec^2 t + 8\sin^2 t\right) dt$$

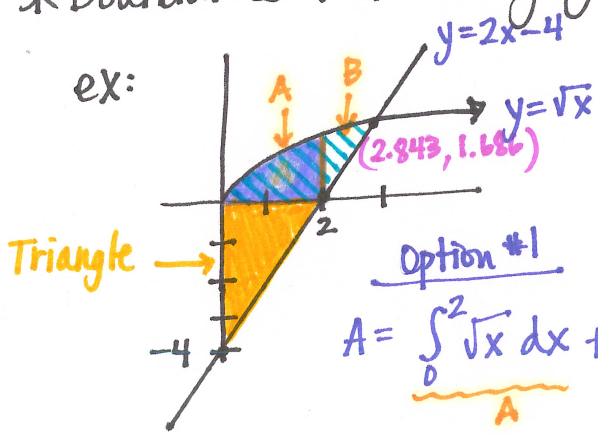
$$= \tan t + 4t - 2\sin 2t \Big|_0^{\pi/3}$$

$$= \left(\tan \tfrac{\pi}{3} + 4\left(\tfrac{\pi}{3}\right) - 2\sin \tfrac{2\pi}{3}\right) - (0)$$

$$= \sqrt{3} + \frac{4\pi}{3} - \sqrt{3} = \boxed{\frac{4\pi}{3}}$$

* Boundaries with Changing Functions:

ex:



Find the area of the striped shade region.

* First: find the point of intersection!
  (2.843, 1.686) * Use the x-value since we have y= equations!

Option #1

$$A = \underbrace{\int_0^2 \sqrt{x}\,dx}_{A} + \underbrace{\int_2^{2.843} \left[(\sqrt{x}) - (2x-4)\right] dx}_{B}$$

$$= \tfrac{2}{3}x^{3/2}\Big|_0^2 + \tfrac{2}{3}x^{3/2} - x^2 + 4x\Big|_2^{2.843}$$

$$= (1.886 - 0) + (.599) \boxed{\approx 2.485}$$

Option #2

(upper − lower) then subtract the triangle

$$\int_0^{2.843} \left[\sqrt{x} - (2x-4)\right] dx - \tfrac{1}{2}(2)(4)$$

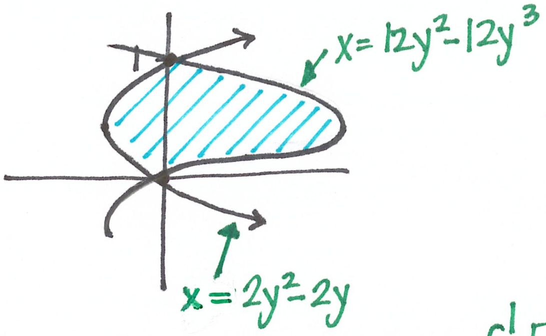
$$\tfrac{2}{3}x^{3/2} - x^2 + 4\Big|_0^{2.843} - 4$$

$$\approx 6.485 - 4$$

same! $\boxed{\approx 2.485}$

* Integrating with Respect to y: right graph − left graph

#4



Find the shaded area.

* Find the points of intersection: (0,1) & (0,0)

* Use the y-value since we have x= equations!

$$A = \int_0^1 \left[\overset{\text{right}}{(12y^2-12y^3)} - \overset{\text{left}}{(2y^2-2y)}\right] dy$$

$$= \int_0^1 (-12y^3+10y^2+2y)\,dy$$

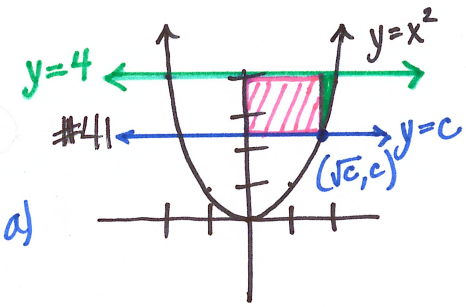
$$= -3y^4+\frac{10}{3}y^3+y^2 \Big|_0^1$$

$$= -3(1)^4+\frac{10}{3}(1)^3+(1)^2-(0)$$

$$= -3+\frac{10}{3}+1 \ \boxed{= \frac{4}{3}}$$

#41

a)



b) $y=x^2$, $x=\pm\sqrt{y}$, $x=\sqrt{y}$ } w/ Respect to y: Right−Left

$$\int_0^c(\sqrt{y}-0)\,dy = \int_c^4(\sqrt{y}-0)\,dy$$

$$\frac{2}{3}y^{3/2}\Big|_0^c = \frac{2}{3}y^{3/2}\Big|_c^4$$

$$\frac{2}{3}c^{3/2}-0 = \frac{2}{3}(8)-\frac{2}{3}c^{3/2}$$

$$\frac{4}{3}c^{3/2} = \frac{16}{3}$$

$$c^{3/2} = 4$$

$$\boxed{c = 4^{2/3} \text{ or } 2^{4/3}}$$

c) w/ Respect to x: Upper−Lower

$$\int_0^{\sqrt{c}}(c-x^2)\,dx = (4-c)\sqrt{c} + \int_{\sqrt{c}}^2(4-x^2)\,dx$$

$$cx-\frac{x^3}{3}\Big|_0^{c^{1/2}} = 4c^{1/2}-c^{3/2}+\left[4x-\frac{x^3}{3}\Big|_{c^{1/2}}^2\right]$$

$$c^{3/2}-\frac{c^{3/2}}{3}-0 = \cancel{4c^{1/2}}-c^{3/2}+8-\frac{8}{3}+\left(\cancel{4c^{1/2}}+\frac{c^{3/2}}{3}\right)$$

$$\frac{2}{3}c^{3/2} = \frac{16}{3}-\frac{2}{3}c^{3/2}$$

$$4c^{3/2} = 16$$

$$c^{3/2} = 4$$

$$\boxed{c = 4^{2/3} \text{ or } 2^{4/3}}$$