Section 8.2 (Day 2)

#19 * Pick an "a" to see what the graph looks like!

$$2\int_0^a x\sqrt{a^2-x^2}\,dx$$

* Do a "u-sub"

$u = a^2-x^2$ $\quad u(0)=a^2$

$du = -2x\,dx$ $\quad u(a)=a^2-a^2=0$

$-\frac{1}{2}du = x\,dx$

$$= 2\int_{a^2}^{0} u^{1/2}\cdot -\frac{1}{2}du$$

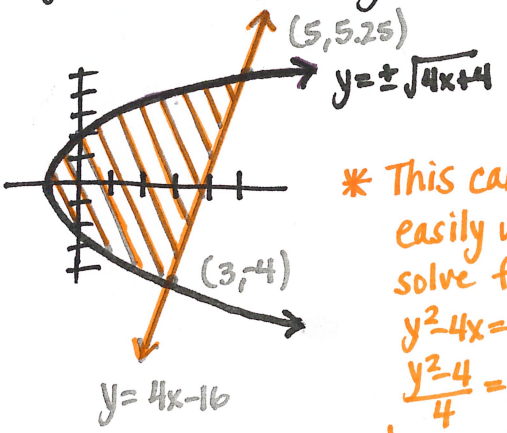
$$= 2\left[-\frac{1}{2}\cdot\frac{2}{3}u^{3/2}\right]\Big|_{a^2}^{0}$$

$$= -\frac{2}{3}\left[0^{3/2}-(a^2)^{3/2}\right] = \boxed{\frac{2}{3}a^3}$$

#23 $y^2-4x=4$ & $4x-y=16$

$\rightarrow$ $y^2=4x+4$ & $y=4x-16$

$y=\pm\sqrt{4x+4}$



* This cannot be integrated easily with dx's, so... solve for x & use dy's & y-values!

$y^2-4x=4$ & $4x-y=16$

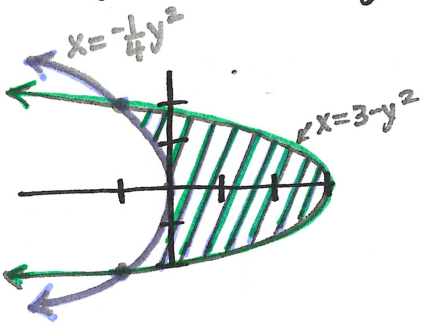
$\frac{y^2-4}{4}=x$ (left graph) $\quad$ $x=\frac{16+y}{4}$ (right graph)

Integrate using right − left

$$\int_{-4}^{5}\left[\frac{16+y}{4}-\left(\frac{y^2-4}{4}\right)\right]dy$$

$$= 30.375$$

#21 $x+y^2=3$ & $4x+y^2=0$



$x=3-y^2$ & $x=-\frac{1}{4}y^2$

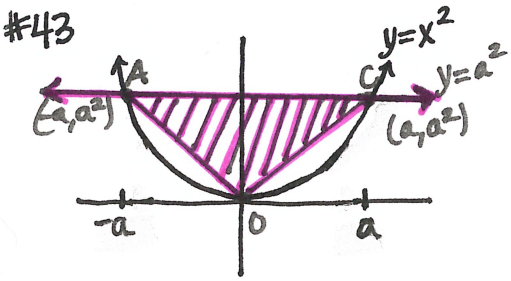
$$\int_{-2}^{2}\left[(3-y^2)-\left(-\frac{1}{4}y^2\right)\right]dy$$

OR $= 2\int_0^2\left[(3-y^2)+\left(\frac{1}{4}y^2\right)\right]dy$

$$= 2\left[3y-\frac{3}{12}y^3\Big|_0^2\right]$$

$$= 2\left[6-\frac{1}{4}(8)-0\right] = 2(6-2) = \boxed{8}$$

#43



Ratio: $\frac{\text{Triangle}}{\text{Parabola}} = \frac{a^3}{4a^3/3} = \boxed{\frac{3}{4}}$

Area of Triangle: $\frac{1}{2}b\cdot h = \frac{1}{2}(2a)(a^2) = a^3$

Area of Parabola: $2\int_0^a (a^2-x^2)\,dx = 2\left(a^2x-\frac{x^3}{3}\right)\Big|_0^a$

$$= 2\left(a^3-\frac{a^3}{3}-0\right)$$

$$= \frac{4a^3}{3}$$