Section 8.2 : 1-45 e.o.o.

1. $\displaystyle\int_0^{\pi} (1-\cos^2 x)\,dx$

$\sin^2 x + \cos^2 x = 1 \rightarrow 1-\cos^2 x = \sin^2 x$

$\cos 2x = 1 - 2\sin^2 x \rightarrow 2\sin^2 x = 1 - \cos 2x$

$\sin^2 x = \frac{1}{2} - \frac{1}{2}\cos 2x$

$$\int_0^{\pi} \sin^2 x\,dx = \int_0^{\pi} \left(\tfrac{1}{2} - \tfrac{1}{2}\cos 2x\right)dx = \left(\tfrac{1}{2}x - \tfrac{1}{4}\sin 2x\right)\Big|_0^{\pi} = \left(\tfrac{1}{2}\pi - 0\right) - (0-0) = \boxed{\frac{\pi}{2}}$$

5. $$2\int_0^2 (2x^2 - x^4 + 2x^2)\,dx = 2\int_0^2 (4x^2 - x^4)\,dx = 2\left(\tfrac{4}{3}x^3 - \tfrac{1}{5}x^5\right)\Big|_0^2 = 2\left(\tfrac{32}{3} - \tfrac{32}{5}\right)$$

$$2\left(\tfrac{160}{15} - \tfrac{96}{15}\right) = 2\cdot\tfrac{64}{15} = \boxed{\frac{128}{15}}$$

9. $y = x$, $x = y$

$y = \frac{1}{4}x^2 \rightarrow x^2 = 4y \rightarrow x = \sqrt{4y} = 2\sqrt{y} = 2y^{1/2}$

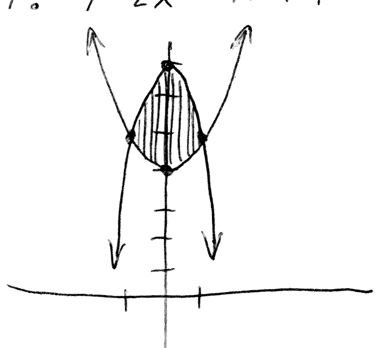
$$\int_0^1 (2y^{1/2} - y)\,dy = \left(2\cdot\tfrac{2}{3}y^{3/2} - \tfrac{1}{2}y^2\right)\Big|_0^1 = \left(\tfrac{4}{3}y^{3/2} - \tfrac{1}{2}y^2\right)\Big|_0^1 = \tfrac{4}{3} - \tfrac{1}{2} = \tfrac{8}{6} - \tfrac{3}{6} = \boxed{\frac{5}{6}}$$

13. $$\int_{-2}^{0} (2x^3 \cancel{-x^2} - 5x \cancel{+x^2} - 3x)\,dx = \int_{-2}^{0} (2x^3 - 8x)\,dx = \left(\tfrac{1}{2}x^4 - 4x^2\right)\Big|_{-2}^{0} = 0 - (8-16) = 8$$

$$\int_0^2 (\cancel{-x^2} + 3x - 2x^3 \cancel{+x^2} + 5x)\,dx = \int_0^2 (8x - 2x^3)\,dx = \left(4x^2 - \tfrac{1}{2}x^4\right)\Big|_0^2 = (16-8) - 0 = 8$$

Total Area $= 8 + 8 = \boxed{16}$

17. $7 - 2x^2 = x^2 + 4 \rightarrow 3 = 3x^2 \rightarrow x^2 = 1 \rightarrow x = \pm 1$ intersection points

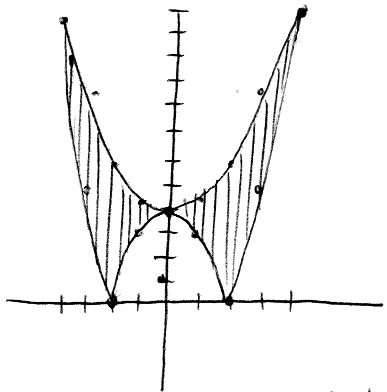


$$2\int_0^1 (7 - 2x^2 - x^2 - 4)\,dx = 2\int_0^1 (3 - 3x^2)\,dx = 2(3x - x^3)\Big|_0^1$$

$$2(3-1) - 2(0) = 2(2) = \boxed{4}$$

21. $|x^2-4| = \frac{1}{2}x^2 + 4$ means $x^2-4 = \frac{1}{2}x^2+4$ or $x^2-4 = -\frac{1}{2}x^2-4$

$$\frac{1}{2}x^2 = 8 \qquad\qquad \frac{3}{2}x^2 = 0$$
$$x^2 = 16 \rightarrow x = \pm 4 \qquad\qquad x^2 = 0 \rightarrow x = 0$$



$$|x^2-4| = \begin{cases} -x^2+4, & -2 \le x \le 2 \\ x^2-4, & x \ge 2 \cup x \le -2 \end{cases}$$

2x Right Side = Total

$$2\int_0^2 \left(\tfrac{1}{2}x^2+4+x^2-4\right)dx = 2\int_0^2 \left(\tfrac{3}{2}x^2\right)dx = 2\cdot\tfrac{1}{2}x^3\Big|_0^2 = x^3\Big|_0^2 = 8-0 = 8$$

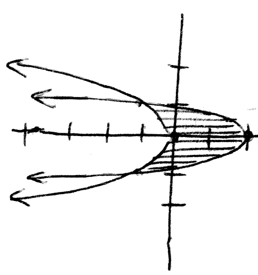
$$2\int_2^4 \left(\tfrac{1}{2}x^2+4-x^2+4\right)dx = 2\int_2^4 \left(-\tfrac{1}{2}x^2+8\right)dx = 2\left(-\tfrac{1}{6}x^3+8x\right)\Big|_2^4 = \left(-\tfrac{1}{3}x^3+16x\right)\Big|_2^4$$

$$= \left(\tfrac{-64}{3}+64\right) - \left(\tfrac{-8}{3}+32\right) = -\tfrac{56}{3}+32 = \tfrac{-56}{3}+\tfrac{96}{3} = \tfrac{40}{3}$$

$$\text{Total Area} = 8 + \tfrac{40}{3} = \tfrac{24}{3} + \tfrac{40}{3} = \boxed{\tfrac{64}{3}}$$

25. $x+y^2 = 0$ and $x+3y^2 = 2$

$$x = -y^2 \qquad x = 2-3y^2 \qquad -y^2 = 2-3y^2$$
$$2y^2 = 2$$
$$y^2 = 1$$
$$y = \pm 1$$



Right - Left

Total = 2x Top Half

$$2\int_0^1 (2-3y^2+y^2)\,dy = 2\int_0^1 (2-2y^2)\,dy = 2\left(2y-\tfrac{2}{3}y^3\right)\Big|_0^1$$

$$2\left(2-\tfrac{2}{3}\right) = 4-\tfrac{4}{3} = \tfrac{12}{3}-\tfrac{4}{3} = \boxed{\tfrac{8}{3}}$$

29. $y = 8\cos x$, $y = \sec^2 x$, $-\frac{\pi}{3} \le x \le \frac{\pi}{3}$

$8\cos x = \sec^2 x$

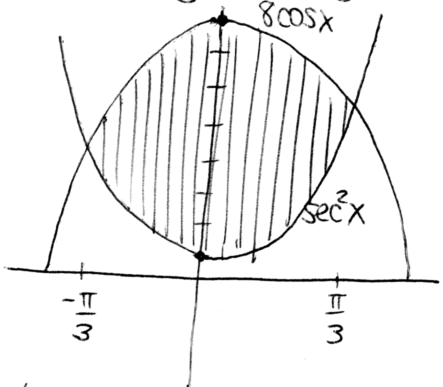
$8\cos x = \frac{1}{\cos^2 x}$

$8\cos^3 x = 1$

$(\cos x)^3 = 1/8$

$\cos x = 1/2$

$x = \frac{\pi}{3}, -\frac{\pi}{3}$



(supposed to be symmetric)

Total = 2 × Right

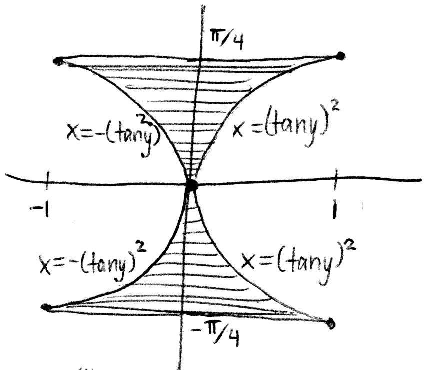
$2\int_0^{\pi/3} (8\cos x - \sec^2 x)\,dx$

$2(8\sin x - \tan x)\Big|_0^{\pi/3}$

$2(8\cdot\sqrt{3}/2 - \sqrt{3}) - 2(0-0)$

$2(4\sqrt{3} - \sqrt{3}) = 2\cdot 3\sqrt{3} = \boxed{6\sqrt{3}}$

33. $x = (\tan y)^2$ and $x = -(\tan y)^2$, $-\frac{\pi}{4} \le y \le \frac{\pi}{4}$



Total = 2 × Top Half

Right − Left

$2\int_0^{\pi/4} ((\tan y)^2 - -(\tan y)^2)\,dy = 2\int_0^{\pi/4} 2(\tan y)^2\,dy$

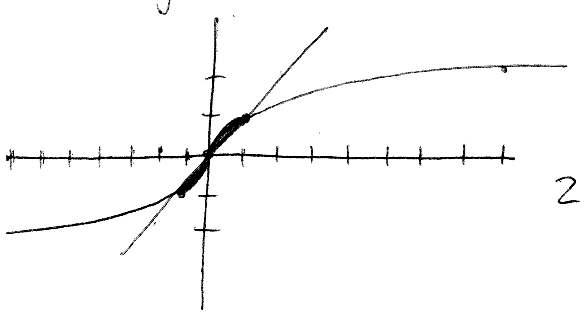
$\sin^2 y + \cos^2 y = 1 \rightarrow \tan^2 y + 1 = \sec^2 y \rightarrow \tan^2 y = \sec^2 y - 1$

$2\int_0^{\pi/4} (2\sec^2 y - 2)\,dy = 2(2\tan y - 2y)\Big|_0^{\pi/4} = 2(2\cdot 1 - 2\cdot\frac{\pi}{4}) - 2(\cancel{0-0}) = 2(2 - \frac{\pi}{2})$

$= \boxed{4 - \pi}$

37. $x = y^3$, $x = y \rightarrow y^3 = y \rightarrow y^3 - y = 0 \rightarrow y(y^2-1) = 0 \rightarrow y(y+1)(y-1) = 0$

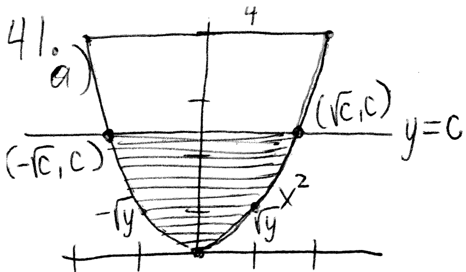
$y = 0, -1, 1$



Total = 2× Right, Right − Left

$2\int_0^1 (y - y^3)\,dy = 2(\frac{1}{2}y^2 - \frac{1}{4}y^4)\Big|_0^1 = 2(\frac{1}{2} - \frac{1}{4}) = 2\cdot\frac{1}{4} = \boxed{\frac{1}{2}}$

41. a)



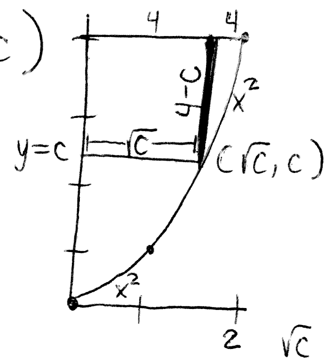
$c = x^2 \to x = \pm\sqrt{c}$ $\quad$ $\boxed{\begin{matrix}(\sqrt{c}, c)\\(-\sqrt{c}, c)\end{matrix}}$ $\quad$ $y = x^2$, $x = \pm\sqrt{y}$

b) $\int_0^c (\sqrt{y} - -\sqrt{y})\,dy = \int_c^4 (\sqrt{y} - -\sqrt{y})\,dy \to \int_0^c 2y^{1/2}\,dy = \int_c^4 2y^{1/2}\,dy$

$\frac{4}{3} y^{3/2}\Big|_0^c = \frac{4}{3} y^{3/2}\Big|_c^4 \to \frac{4}{3} c^{3/2} - 0 = \frac{4}{3}\cdot 4^{3/2} - \frac{4}{3} c^{3/2}$

$\frac{8}{\cancel{3}} c^{3/2} = \frac{4}{\cancel{3}}\cdot 8 \to 8c^{3/2} = 32 \to c^{3/2} = 4 \to \boxed{c = 4^{2/3}}$

c)



Bottom = Top

$\int_0^{\sqrt{c}} (c - x^2)\,dx = \sqrt{c}(4-c) + \int_{\sqrt{c}}^2 (4 - x^2)\,dx$

$(cx - \frac{1}{3}x^3)\Big|_0^{\sqrt{c}} = 4c^{1/2} - c^{3/2} + (4x - \frac{1}{3}x^3)\Big|_{\sqrt{c}}^2$

$c^{3/2} - \frac{1}{3}c^{3/2} = \cancel{4c^{1/2}} - c^{3/2} + (8 - 8/3) - (\cancel{4c^{1/2}} - \frac{1}{3}c^{3/2})$

$\frac{2}{3}c^{3/2} = 8 - \frac{8}{3} - \frac{2}{3}c^{3/2} \to \frac{4}{\cancel{3}}c^{3/2} = \frac{16}{\cancel{3}} \to 4c^{3/2} = 16 \to c^{3/2} = 4 \to \boxed{c = 4^{2/3}}$

15. i) $\int_{-1}^{1} 2x\,dx = x^2\Big|_{-1}^{1} = 1 - 1 = 0 \to$ not true

ii) $\int_{-1}^{1} -2x\,dx = -x^2\Big|_{-1}^{1} = -1 + 1 = 0 \to$ not true

Correct: $\int_{-1}^{0} (-x - x)\,dx + \int_0^1 (x - -x)\,dx = \int_{-1}^{0} -2x\,dx + \int_0^1 2x\,dx$

$-x^2\Big|_{-1}^{0} + x^2\Big|_0^1 = 0 + 1 + 1 - 0 = \boxed{2}$

Triangles: $\cancel{2}\cdot\frac{1}{\cancel{2}}\cdot 2 \cdot 1 = 2$ ✓