Section 8.2: 3–43 e.o.o.

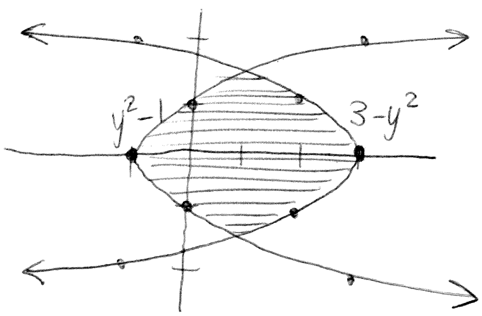
3. $\displaystyle\int_0^1 (y^2-y^3)\,dy = \left(\tfrac{1}{3}y^3-\tfrac{1}{4}y^4\right)\Big|_0^1 = \frac{1}{3}-\frac{1}{4} = \frac{4}{12}-\frac{3}{12} = \boxed{\frac{1}{12}}$

7. $y=\sin x$, $y=1-x^2$

Intersect at $x=-1.409624$, $x=0.63673265$

$\text{NINT}(1-x^2-\sin x, x, -1.409624, 0.63673265) = \boxed{1.670}$

11. $x=y^2-1$, $x=3-y^2$



$y^2-1=3-y^2 \rightarrow 2y^2=4 \rightarrow y^2=2 \rightarrow y=\pm\sqrt{2}$

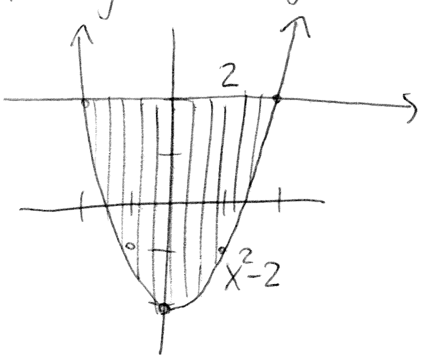
Total = 2 × Top Half

$\displaystyle 2\int_0^{\sqrt{2}} (3-y^2-y^2+1)\,dy = 2\int_0^{\sqrt{2}} (4-2y^2)\,dy$

$\displaystyle 2\left(4y-\tfrac{2}{3}y^3\right)\Big|_0^{\sqrt{2}} = 8\sqrt{2}-\frac{4}{3}\cdot\sqrt{2}^3 = 8\sqrt{2}-\frac{4}{3}\cdot 2\sqrt{2}$

$\displaystyle = \left(8-\frac{8}{3}\right)\sqrt{2} = \boxed{\frac{16}{3}\sqrt{2}}$

15. $y=x^2-2$, $y=2$

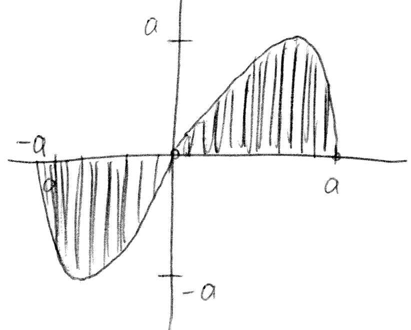


Total = 2 × Right Side

$\displaystyle 2\int_0^2 (2-x^2+2)\,dx = 2\int_0^2 (4-x^2)\,dx = 2\left(4x-\tfrac{1}{3}x^3\right)\Big|_0^2$

$\displaystyle 2\left(8-\frac{8}{3}\right) = 2\cdot\frac{16}{3} = \boxed{\frac{32}{3}}$

19. $y=x\sqrt{a^2-x^2}$, $a>0$ and $y=0$ (Use a few values of a to see the general shape of the graph.)



(supposed to be symmetric)

Total = 2 × Right = $\displaystyle 2\int_0^a x\sqrt{a^2-x^2}\,dx$

$u=a^2-x^2$

$du=-2x\,dx$

$dx=\dfrac{du}{-2x}$

$\displaystyle 2\int \cancel{x}\sqrt{u}\cdot\frac{du}{-2\cancel{x}} = 2\cdot\int -\tfrac{1}{2}u^{1/2}\,du = \cancel{2}\cdot\frac{-1}{\cancel{2}}\int u^{1/2}\,du$

$\displaystyle -\frac{2}{3}u^{3/2} = -\frac{2}{3}(a^2-x^2)^{3/2}\Big|_0^a$

$\displaystyle -\frac{2}{3}(a^2-a^2)^{3/2} + \frac{2}{3}(a^2-0)^{3/2} = 0+\frac{2}{3}(a^2)^{3/2} = \boxed{\frac{2}{3}a^3}$

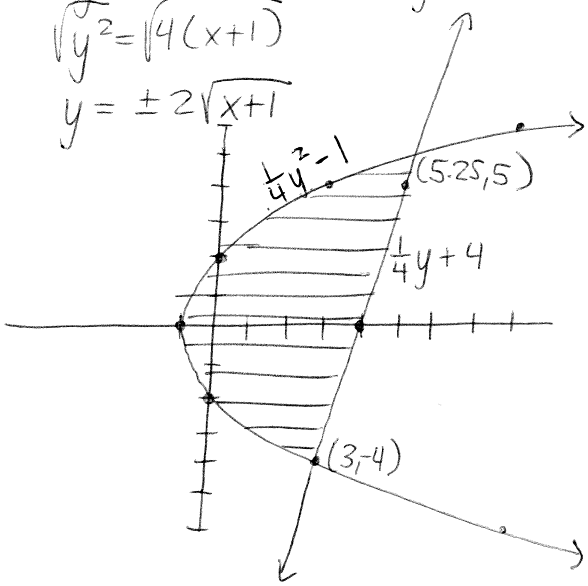
23. $y^2 - 4x = 4$ and $4x - y = 16$

$y^2 = 4x + 4$

$\sqrt{y^2} = \sqrt{4(x+1)}$

$y = \pm 2\sqrt{x+1}$

$y = 4x - 16$



$y^2 - 4x = 4$ | $4x - y = 16$
$4x = y^2 - 4$ | $4x = y + 16$
$x = \frac{1}{4}y^2 - 1$ | $x = \frac{1}{4}y + 4$

Right - Left

$$\int_{-4}^{5}\left(\tfrac{1}{4}y + 4 + 1 - \tfrac{1}{4}y^2\right)dy = \left(\tfrac{1}{8}y^2 + 5y - \tfrac{1}{12}y^3\right)\Big|_{-4}^{5}$$

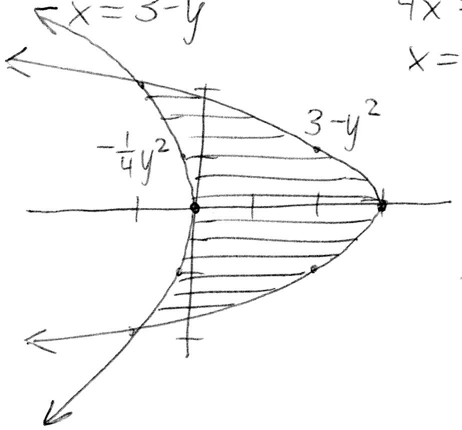
$$\frac{425}{24} - -\frac{38}{3} = \frac{425}{24} + \frac{304}{24} = \frac{729}{24} = \boxed{30.375}$$

27. $x + y^2 = 3$ and $4x + y^2 = 0$

$x = 3 - y^2$

$4x = -y^2$

$x = -\frac{1}{4}y^2$



$3 - y^2 = -\frac{1}{4}y^2$

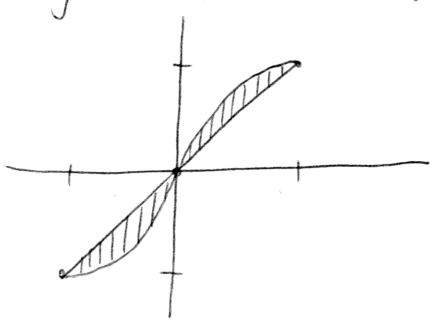
$3 = \frac{3}{4}y^2 \to y^2 = 4 \to y = \pm 2$

Right - Left

$$\text{Total} = 2\times \text{Top Half} = 2\int_0^2 \left(3 - y^2 + \tfrac{1}{4}y^2\right)dy$$

$$2\int_0^2\left(3 - \tfrac{3}{4}y^2\right)dy = 2\left(3y - \tfrac{1}{4}y^3\right)\Big|_0^2 = 2(6-2) = \boxed{8}$$

31. $y = \sin(\frac{\pi}{2}x)$ and $y = x$ intersect at $x = 0, 1, -1$



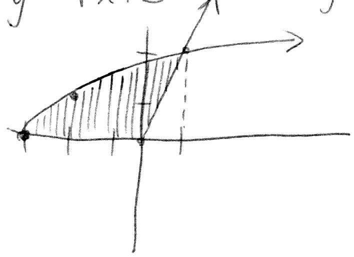
Top - Bottom

Total = 2 × Top Half

$$2\int_0^1\left(\sin(\tfrac{\pi}{2}x) - x\right)dx = 2\left(-\tfrac{2}{\pi}\cos(\tfrac{\pi}{2}x) - \tfrac{1}{2}x^2\right)\Big|_0^1$$

$$2(\cancel{0} - \tfrac{1}{2}) - 2(-\tfrac{2}{\pi} - \cancel{0}) = \boxed{-1 + \frac{4}{\pi}}$$

35. $y=\sqrt{x+3}$ and $y=2x$, above x-axis (so not using $y=-\sqrt{x+3}$)



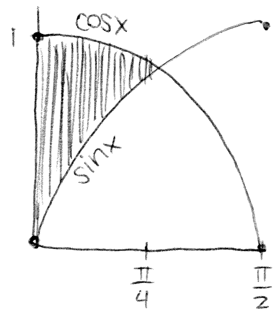
Total with triangle included:

$$\int_{-3}^{1}(x+3)^{1/2}dx = \frac{2}{3}(x+3)^{3/2}\Big|_{-3}^{1} = \frac{2}{3}\cdot 4^{3/2} - \cancel{\frac{2}{3}\cdot 0^{3/2}} = \frac{2}{3}\cdot 8 = \frac{16}{3}$$

Triangle $= \frac{1}{2}bh = \frac{1}{2}(1)(2) = 1$

Shaded = Area under curve - triangle $= \frac{16}{3} - 1 = \frac{16}{3} - \frac{3}{3} = \boxed{\frac{13}{3}}$

39.



$$\int_{0}^{\pi/4}(\cos x - \sin x)dx = (\sin x + \cos x)\Big|_{0}^{\pi/4}$$

$$\left(\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}\right) - (0+1) = \boxed{\sqrt{2}-1}$$

43. Triangle $= \frac{1}{2}bh = \cancel{\frac{1}{2}}\cdot\cancel{2}a\cdot a^2 = a^3$

Curved Region $= 2\int_{0}^{a}(a^2-x^2)dx = 2\left(a^2x - \frac{1}{3}x^3\right)\Big|_{0}^{a}$

$$= 2\left(a^3 - \frac{1}{3}a^3\right) - \cancel{2(0)} = 2\cdot\frac{2}{3}a^3 = \frac{4}{3}a^3$$

Ratio of Δ Area to Curved Area $= \frac{\cancel{a^3}}{\frac{4}{3}\cancel{a^3}} = \frac{1}{4/3} = \boxed{\frac{3}{4}}$