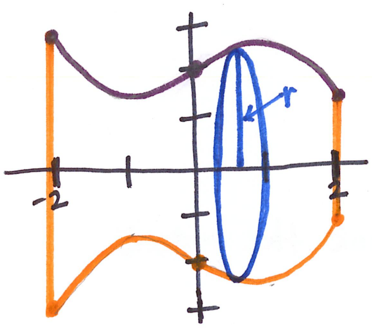
Volumes (Section 8.3) Day 2

* Volume: $V = \int_a^b A(x)\,dx$ ← Area

* Slicing Method:
  1. Sketch the solid & a typical cross section.
  2. Find a formula for $A(x)$.
  3. Find the limits of integration
  4. Integrate!

ex: Revolve $f(x) = 2 + x\cos x$ around the x-axis on $[-2, 2]$.



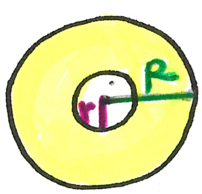
$A = \pi r^2 \longrightarrow \pi(2 + x\cos x)^2$

$r = $ y-value $\rightarrow f(x)$

$\therefore V = \int_{-2}^{2} \pi(2 + x\cos x)^2\,dx$ ← use the calc!

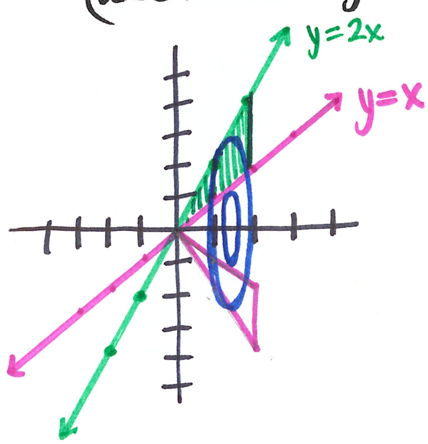
$\boxed{= 52.429 \text{ units}^3}$

* Washer Cross Sections: used when there is a hole (or gap) in the middle of the cross section.



Area of a washer $= \pi R^2 - \pi r^2$
$= \pi(R^2 - r^2)$

ex: Using $y = 2x$ & $y = x$, revolve about the x-axis.
(and bounded by $x = 2$)



*Use washer method!

$R = 2x$
$r = x$

$A(x) = \pi[(2x)^2 - (x)^2]$
$= \pi(4x^2 - x^2) = \pi(3x^2)$

$V = \int_0^2 3\pi x^2\,dx$

$= \frac{\cancel{3}\pi x^3}{\cancel{3}}\Big|_0^2$

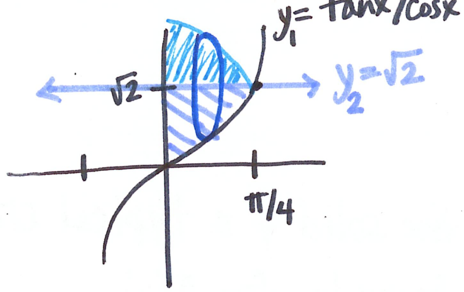
$= \pi(8 - 0) = \boxed{8\pi}$

#21 $y = \sec x \cdot \tan x$

$y = \sqrt{2}$

y-axis

About $y = \sqrt{2}$



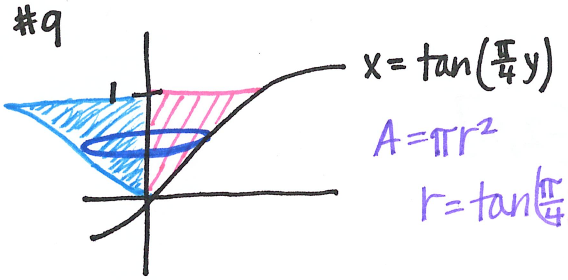
$A = \pi r^2$

$r = \sqrt{2} - \underbrace{\sec x \tan x}_{\frac{\tan x}{\cos x}}$

$$\int_0^{\pi/4} \pi \left(\sqrt{2} - \frac{\tan x}{\cos x} \right)^2 dx$$

OR $\displaystyle\int_0^{\pi/4} \pi (y_2 - y_1)^2 dx = \boxed{2.301}$

#9



about the y-axis

$A = \pi r^2$

$r = \tan(\frac{\pi}{4} y)$

$$\int_0^1 \pi \left[\tan\left(\tfrac{\pi}{4} y\right) \right]^2 dy$$

"a" in the table

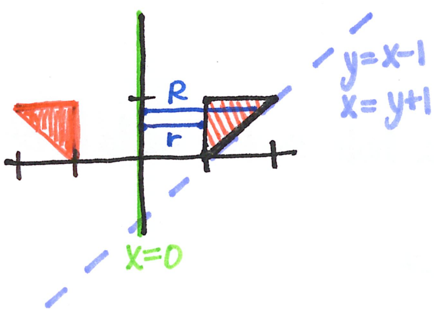
*To do this by hand, you must use the tables in your book. (pg 623 #84)

$$= \pi \left[\left(\tfrac{4}{\pi} \tan \tfrac{\pi}{4} y - y \right) \right] \Big|_0^1$$

$$= \pi \left[\left(\tfrac{4}{\pi} \cancelto{1}{\tan \tfrac{\pi}{4}} - 1 \right) - \left(\tfrac{4}{\pi} \cancelto{0}{\tan 0} - 0 \right) \right] = \pi \left(\tfrac{4}{\pi} - 1 \right) = \boxed{4 - \pi}$$

#25 Triangle w/ vertices at (1,0) (2,1) and (1,1) revolved about the y-axis.

↳ x=0 line



*Washer Method!

$R = y + 1$

$r = 1$

$$A = \pi \left[(y+1)^2 - (1)^2 \right]$$

$$= \pi \left[y^2 + 2y + \cancel{1} - \cancel{1} \right]$$

$$= \pi (y^2 + 2y)$$

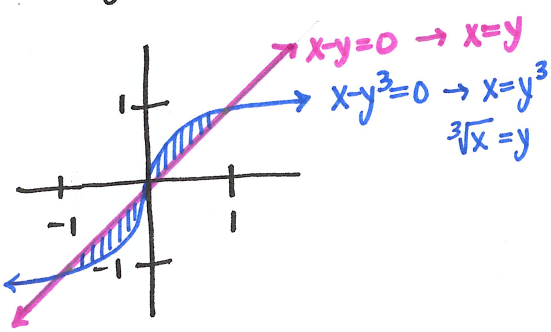
$$\int_0^1 \pi (y^2 + 2y)\, dy$$

$$= \pi \left(\frac{y^3}{3} + y^2 \right) \Big|_0^1 = \pi \left[\left(\tfrac{1}{3} + 1 \right) - 0 \right] = \boxed{\frac{4\pi}{3}}$$

Section 8.2!

#37 $x - y^3 = 0$ Find the area enclosed.

$x - y = 0$



$$2 \int_0^1 \left(\sqrt[3]{x} - x \right) dx = 2 \int_0^1 \left(x^{1/3} - x \right) dx$$

$$= 2 \left(\frac{3}{4} x^{4/3} - \frac{x^2}{2} \right) \Big|_0^1$$

$$= 2 \left(\frac{3}{4} - \frac{1}{2} \right) - 0$$

$$\boxed{= \frac{1}{2}}$$