Section 8.3 : 1-6 all

1.a) Disk $= \pi r^2$

$r = \sqrt{1-x^2}$

Disk $= \pi\sqrt{1-x^2}^2 = \boxed{\pi(1-x^2)}$

b) Square $= s^2$

$s = 2\sqrt{1-x^2}$

Square $= (2\sqrt{1-x^2})^2 = 2^2\cdot\sqrt{1-x^2}^2 = \boxed{4(1-x^2)}$

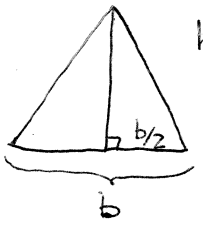
c) Square $= s^2$, Diagonal $= \sqrt{2}\cdot$ Side, Side $= \frac{\text{Diagonal}}{\sqrt{2}}$

Diagonal $= 2\sqrt{1-x^2}$

Side $= \frac{2\sqrt{1-x^2}}{\sqrt{2}} = \frac{2\sqrt{1-x^2}\cdot\sqrt{2}}{2} = \sqrt{2}\cdot\sqrt{1-x^2} = \sqrt{2-2x^2}$

Square $= \sqrt{2-2x^2}^2 = 2-2x^2 = \boxed{2(1-x^2)}$

d)



$h = \frac{\sqrt{3}}{2}b$

Triangle $= \frac{1}{2}bh = \frac{1}{2}\cdot b\cdot\frac{\sqrt{3}}{2}b = \frac{\sqrt{3}}{4}b^2$

Base $= 2\sqrt{1-x^2}$

Triangle $= \frac{\sqrt{3}}{4}\cdot(2\sqrt{1-x^2})^2 = \frac{\sqrt{3}}{4}\cdot 4(1-x^2) = \boxed{\sqrt{3}(1-x^2)}$

2. a) Disk $= \pi r^2$

$r = \sqrt{x}$

Disk $= \pi\cdot\sqrt{x}^2 = \boxed{\pi x}$

b) Square $= s^2$

$s = 2\sqrt{x}$

Square $= (2\sqrt{x})^2 = \boxed{4x}$

c) Square $= s^2$, $s =$ diag$/\sqrt{2}$

Diagonal $= 2\sqrt{x}$

Side $= \frac{2\sqrt{x}\cdot\sqrt{2}}{2} = \sqrt{x}\cdot\sqrt{2} = \sqrt{2x}$

Square $= (\sqrt{2x})^2 = \boxed{2x}$

d) $\Delta = \frac{1}{2}bh = \frac{\sqrt{3}}{4}b^2$ (from #1)

$b = 2\sqrt{x}$

Triangle $= \frac{\sqrt{3}}{4}(2\sqrt{x})^2 = \frac{\sqrt{3}}{4}\cdot 4\cdot x = \boxed{\sqrt{3}x}$

3. From #2c, area of each square = $2x$

$$\int_0^4 2x\,dx = x^2\Big|_0^4 = 16-0 = \boxed{16}$$

4. Disk = πr^2

$d = 2-x^2-x^2 = 2-2x^2$ (top - bottom)

$r = \frac{1}{2}d = 1-x^2$

Disk = $\pi(1-x^2)^2 = \pi(1-2x^2+x^4)$

$$\pi\int_{-1}^{1}(1-2x^2+x^4)dx = \pi\left(x-\frac{2}{3}x^3+\frac{1}{5}x^5\right)\Big|_{-1}^{1} = \pi\left(1-\frac{2}{3}+\frac{1}{5}\right)-\pi\left(-1+\frac{2}{3}-\frac{1}{5}\right)$$

$$= \pi\left(1-\frac{2}{3}+\frac{1}{5}+1-\frac{2}{3}+\frac{1}{5}\right)$$

$$= \pi\left(2-\frac{4}{3}+\frac{2}{5}\right) = \pi\left(\frac{30}{15}-\frac{20}{15}+\frac{6}{15}\right) = \boxed{\frac{16}{15}\pi}$$

5. From #1b, area of each square = $4(1-x^2)$

$$\int_{-1}^{1}(4-4x^2)dx = \left(4x-\frac{4}{3}x^3\right)\Big|_{-1}^{1} = \left(4-\frac{4}{3}\right)-\left(-4+\frac{4}{3}\right) = 8-\frac{8}{3} = \boxed{\frac{16}{3}}$$

6. From #1c, area of each square = $2(1-x^2)$

$$\int_{-1}^{1}(2-2x^2)dx = \left(2x-\frac{2}{3}x^3\right)\Big|_{-1}^{1} = \left(2-\frac{2}{3}\right)-\left(-2+\frac{2}{3}\right) = 4-\frac{4}{3} = \boxed{\frac{8}{3}}$$