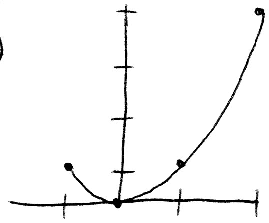
Section 8.4: 1-31 odd

1. $y = x^2$, $-1 \le x \le 2$

b)



a) $\frac{dy}{dx} = 2x$

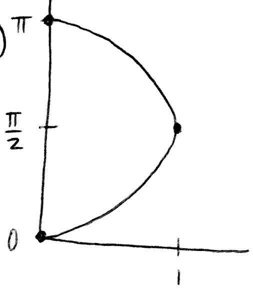
$\left(\frac{dy}{dx}\right)^2 = 4x^2$

$$\boxed{\int_{-1}^{2} \sqrt{1+4x^2}\,dx}$$

c) $\boxed{6.126}$ (NINT)

3. $x = \sin y$, $0 \le y \le \pi$

b)



a) $\frac{dx}{dy} = \cos y$

$\left(\frac{dx}{dy}\right)^2 = \cos^2 y$

$$\boxed{\int_{0}^{\pi} \sqrt{1+\cos^2 y}\,dy}$$

c) $\boxed{3.820}$ (NINT)

5. $y^2 + 2y = 2x + 1$ from $(-1,-1)$ to $(7,3)$

$2x = y^2 + 2y - 1$

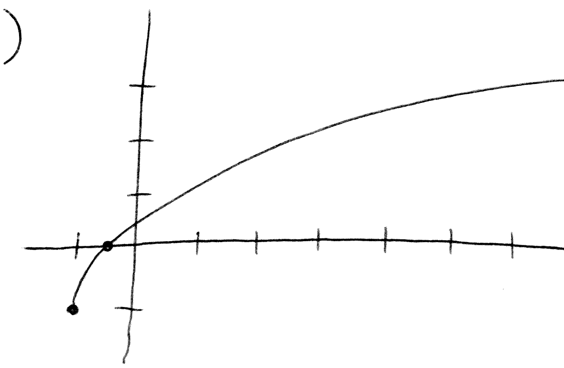
$x = \frac{1}{2}y^2 + y - \frac{1}{2}$

$\frac{dx}{dy} = y + 1$

$\left(\frac{dx}{dy}\right)^2 = (y+1)^2 = y^2 + 2y + 1$

a) $$\int_{-1}^{3} \sqrt{1 + y^2 + 2y + 1}\,dy = \boxed{\int_{-1}^{3} \sqrt{y^2 + 2y + 2}\,dy}$$

b)



c) $\boxed{9.294}$ (NINT)

7. $y = \int_0^x \tan t\,dt$, $0 \le x \le \pi/6$

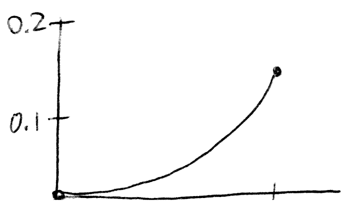
$\frac{dy}{dx} = \tan x$

a) $$\boxed{\int_{0}^{\pi/6} \sqrt{1+\tan^2 x}\,dx}$$

b) $\int \tan x = -\ln(\cos x)$

$-\ln(\cos 0) = -\ln(1) = 0 \rightarrow (0,0)$

$-\ln(\cos \pi/6) = -\ln(\sqrt{3}/2) = 0.144 \rightarrow (\pi/6, 0.144)$



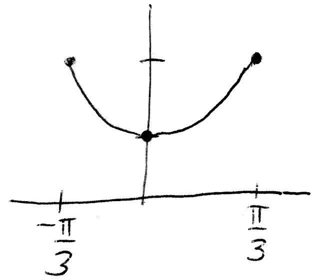
c) $\boxed{0.549}$ (NINT)

9. $y = \sec x, \; -\pi/3 \le x \le \pi/3$

a) $\frac{dy}{dx} = \sec x \tan x$

$$\boxed{\int_{-\pi/3}^{\pi/3} \sqrt{1+\sec^2 x \tan^2 x}\,dx}$$

b)



c) $\boxed{3.139}$ (NINT)

11. $y = \frac{1}{3}(x^2+2)^{3/2}$ from $x=0$ to $x=3$

$$\frac{dy}{dx} = \frac{1}{\cancel{2}}(x^2+2)^{1/2}\cdot\cancel{2}x = x\sqrt{x^2+2}$$

$$\left(\frac{dy}{dx}\right)^2 = \left(x\sqrt{x^2+2}\right)^2 = x^2(x^2+2) = x^4+2x^2$$

$$1+\left(\frac{dy}{dx}\right)^2 = x^4+2x^2+1 = (x^2+1)^2$$

$$\int_0^3 \cancel{\sqrt{(x^2+1)^{\cancel{2}}}}\,dx = \int_0^3 (x^2+1)\,dx = \left(\frac{1}{3}x^3+x\right)\Big|_0^3 = 9+3 = \boxed{12}$$

13. $x = \frac{1}{3}y^3 + \frac{1}{4}y^{-1}$ from $y=1$ to $y=3$

$$\frac{dx}{dy} = y^2 - \frac{1}{4}y^{-2}$$

$$\left(\frac{dx}{dy}\right)^2 = \left(y^2-\frac{1}{4y^2}\right)\left(y^2-\frac{1}{4y^2}\right) = y^4 - \frac{1}{2} + \frac{1}{16y^4}$$

$$1+\left(\frac{dx}{dy}\right)^2 = 1+y^4-\frac{1}{2}+\frac{1}{16y^4} = y^4+\frac{1}{2}+\frac{1}{16y^4} = \left(y^2+\frac{1}{4y^2}\right)^2$$

$$\int_1^3 \cancel{\sqrt{\left(y^2+\frac{1}{4y^2}\right)^{\cancel{2}}}}\,dy = \int_1^3 \left(y^2+\frac{1}{4}y^{-2}\right)dy = \left(\frac{1}{3}y^3-\frac{1}{4}y^{-1}\right)\Big|_1^3$$

$$\left(9-\frac{1}{12}\right)-\left(\frac{1}{3}-\frac{1}{4}\right) = \frac{108}{12}-\frac{1}{12}-\frac{4}{12}+\frac{3}{12} = \frac{106}{12} = \boxed{\frac{53}{6}}$$

15. $x=\frac{1}{6}y^3+\frac{1}{2}y^{-1}$ from $y=1$ to $y=2$

$$\frac{dx}{dy}=\frac{1}{2}y^2-\frac{1}{2}y^{-2}$$

$$\left(\frac{dx}{dy}\right)^2=\left(\frac{1}{2}y^2-\frac{1}{2y^2}\right)\left(\frac{1}{2}y^2-\frac{1}{2y^2}\right)=\frac{1}{4}y^4-\frac{1}{2}+\frac{1}{4y^4}$$

$$1+\left(\frac{dx}{dy}\right)^2=1+\frac{1}{4}y^4-\frac{1}{2}+\frac{1}{4y^4}=\frac{1}{4}y^4+\frac{1}{2}+\frac{1}{4y^4}=\left(\frac{1}{2}y^2+\frac{1}{2y^2}\right)^2$$

$$\int_1^2\cancel{\sqrt{\vphantom{|}}}\left(\frac{1}{2}y^2+\frac{1}{2y^2}\right)^{\cancel{2}}dy=\int_1^2\left(\frac{1}{2}y^2+\frac{1}{2}y^{-2}\right)dy=\left(\frac{1}{6}y^3-\frac{1}{2}y^{-1}\right)\Big|_1^2$$

$$\left(\frac{8}{6}-\frac{1}{4}\right)-\left(\frac{1}{6}-\frac{1}{2}\right)=\frac{32}{24}-\frac{6}{24}-\frac{4}{24}+\frac{12}{24}=\frac{34}{24}=\boxed{\frac{17}{12}}$$

17. $x=\int_0^y\sqrt{\sec^4t-1}\,dt\ ,\ -\pi/4\le y\le\pi/4$

$$\frac{dx}{dy}=\sqrt{\sec^4y-1}$$

$$\left(\frac{dx}{dy}\right)^2=\sec^4y-1$$

$$1+\left(\frac{dx}{dy}\right)^2=\cancel{1}+\sec^4y-\cancel{1}=\sec^4y$$

$$\int_{-\pi/4}^{\pi/4}\sqrt{\sec^4y}\,dy=\int_{-\pi/4}^{\pi/4}\sec^2y\,dy=\tan y\Big|_{-\pi/4}^{\pi/4}=1-(-1)=\boxed{2}$$

19. $L=\int_1^4\sqrt{1+\frac{1}{4x}}\,dx$

a) $\frac{1}{4x}=\left(\frac{dy}{dx}\right)^2\rightarrow\ \frac{dy}{dx}=\sqrt{\frac{1}{4x}}=\frac{1}{2\sqrt{x}}=\frac{1}{2}x^{-1/2}$

$$y=\int\frac{1}{2}x^{-1/2}dx=x^{1/2}+C=\sqrt{x}+C$$

Through $(1,1)$: $1=\sqrt{1}+C\rightarrow\ C=0\rightarrow\ \boxed{y=\sqrt{x}}$

b) $y=-\sqrt{x}=-x^{1/2}$ has $\frac{dy}{dx}=-\frac{1}{2}x^{-1/2}=\frac{-1}{2\sqrt{x}}$, so $\left(\frac{dy}{dx}\right)^2=\left(\frac{-1}{2\sqrt{x}}\right)^2=\frac{1}{4x}$ also.

Therefore, $\boxed{\text{two}}$ possible curves: $y=\sqrt{x}$ and $y=-\sqrt{x}$.

21. $y=\int_0^x \sqrt{\cos 2t}\, dt$ from $x=0$ to $x=\frac{\pi}{4}$

$$\frac{dy}{dx} = \sqrt{\cos 2x}$$

$$\left(\frac{dy}{dx}\right)^2 = \sqrt{\cos 2x}^{\,2} = \cos 2x$$

$$1+\left(\frac{dy}{dx}\right)^2 = 1+\cos 2x$$

$$\int_0^{\pi/4} \sqrt{1+\cos 2x}\, dx$$

↑ Trig identity: $\cos 2x = 2\cos^2 x - 1$

$$\int_0^{\pi/4} \sqrt{\cancel{1}+2\cos^2 x \cancel{-1}}\, dx = \int_0^{\pi/4} \sqrt{2\cos^2 x}\, dx = \int_0^{\pi/4} \sqrt{2}\cos x\, dx = \sqrt{2}\sin x\Big|_0^{\pi/4} = \sqrt{2}\cdot\frac{1}{\sqrt{2}} = \boxed{1}$$

23. $y=\sin\left(\frac{3\pi}{20}x\right)$, $0 \le x \le 20$

$$\frac{dy}{dx} = \frac{3\pi}{20}\cos\left(\frac{3\pi}{20}x\right)$$

$$\int_0^{20} \sqrt{1+\left(\frac{3\pi}{20}\cos\left(\frac{3\pi}{20}x\right)\right)^2}\, dx = \boxed{21.068 \text{ in}} \quad (\text{NINT})$$

25. $f(x)= x^{1/3}+x^{2/3}$, $0 \le x \le 2$

$f'(x) = \frac{1}{3}x^{-2/3} + \frac{2}{3}x^{-1/3} = \frac{1}{3x^{2/3}} + \frac{2}{3x^{1/3}}$ DNE (vertical slope at $x=0$)

$$y = x^{1/3}+x^{2/3} = x^{1/3}+(x^{1/3})^2$$

$$(x^{1/3})^2 + x^{1/3} - y = 0$$

Quadratic formula with $a=1$, $b=1$, $c=-y$. Solving for $x^{1/3}$ instead of x.

$$x^{1/3} = \frac{-1 \pm \sqrt{1^2-4(1)(-y)}}{2(1)} = \frac{-1\pm\sqrt{1+4y}}{2} = \frac{1}{2}\left(-1+\sqrt{1+4y}\right)$$

$$(x^{1/3})^3 = \left(\frac{1}{2}\left(-1+\sqrt{1+4y}\right)\right)^3 \to x = \frac{1}{8}\left(-1+\sqrt{1+4y}\right)^3$$

$$\frac{dx}{dy} = \frac{3}{8}\left(-1+\sqrt{1+4y}\right)^2\cdot\frac{1}{2}(1+4y)^{-1/2}\cdot 4 = \frac{3\left(-1+\sqrt{1+4y}\right)^2}{4\sqrt{1+4y}}$$

$$\int_0^{2^{1/3}+2^{2/3}} \sqrt{1+\left(\frac{3\left(-1+\sqrt{1+4y}\right)^2}{4\sqrt{1+4y}}\right)^2}\, dy = \boxed{3.614}$$

(NINT)

27. $y = x^3 + 5|x|$ from $x = -2$ to $x = 1$

$$y = \begin{cases} x^3 + 5x, & x \geq 0 \\ x^3 - 5x, & x < 0 \end{cases}$$

$$\frac{dy}{dx} = \begin{cases} 3x^2 + 5, & x \geq 0 \\ 3x^2 - 5, & x < 0 \end{cases}$$

$$\int_{-2}^{0} \sqrt{1 + (3x^2 - 5)^2}\, dx + \int_{0}^{1} \sqrt{1 + (3x^2 + 5)^2}\, dx = \boxed{13.132}$$

(NINT)

29. $y = \sqrt[4]{x} = x^{1/4}$ from $x = 0$ to $x = 16$

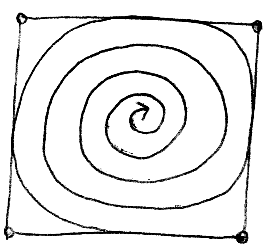
$\frac{dy}{dx} = \frac{1}{4}x^{-3/4} = \frac{1}{4x^{3/4}}$ DNE at $x = 0$ (vertical tangent)

$(y)^4 = (x^{1/4})^4 \rightarrow x = y^4$

$\frac{dx}{dy} = 4y^3$

$y = \sqrt[4]{0} = 0$, $y = \sqrt[4]{16} = 2$ (bounds)

$$\int_0^2 \sqrt{1 + (4y^3)^2}\, dy = \int_0^2 \sqrt{1 + 16y^6}\, dy = \boxed{16.647}$$

31. 

No limit bc the curve could continue infinitely within that area, such as a spiral.