Section 9.1: 1–41 e.o.o., 45–48 all, 55

1. $a_n = \frac{n}{n+1}$

$a_1 = \frac{1}{1+1} = \boxed{\frac{1}{2}}$, $a_2 = \frac{2}{2+1} = \boxed{\frac{2}{3}}$, $a_3 = \frac{3}{3+1} = \boxed{\frac{3}{4}}$, $a_4 = \frac{4}{4+1} = \boxed{\frac{4}{5}}$,

$a_5 = \frac{5}{5+1} = \boxed{\frac{5}{6}}$, $a_6 = \frac{6}{6+1} = \boxed{\frac{6}{7}}$, $a = \frac{50}{50+1} = \boxed{\frac{50}{51}}$

5. $a_1 = 3$, $a_n = a_{n-1} - 2$

$\boxed{3, 1, -1, -3}, \ldots, \boxed{-11}$

9. $U_1 = 1$, $U_2 = 1$, $U_n = U_{n-1} + U_{n-2}$

$\boxed{1, 1, 2, 3,}\ 5, 8, 13, \boxed{21}$

3. $1, 1.5, 2, 2.5, \ldots$

a) $1.5 - 1 = \boxed{0.5}$ or $1/2$

b) $1 + (8-1)(1/2) = 1 + 7(1/2) = 1 + 3.5 = \boxed{4.5}$

c) $a_n = \boxed{a_{n-1} + 0.5}$

d) $a_n = 1 + (n-1)0.5 = 1 + 0.5n - 0.5 = \boxed{0.5n + 0.5}$

17. $-3, 9, -27, 81$

a) $9/{-3} = \boxed{-3}$

b) $-3(-3)^{9-1} = -3(-3)^8 = (-3)^9 = \boxed{-19{,}683}$

c) $a_n = \boxed{a_{n-1}(-3)}$

d) $a_n = -3(-3)^{n-1} = (-3)^1(-3)^{n-1} = (-3)^{1+n-1} = \boxed{(-3)^n}$

21. ___, ___, ___, 3,010, ___, ___, 3,010,000

$\div r \quad \div r \quad \div r \qquad \times r \quad \times r \quad \times r$

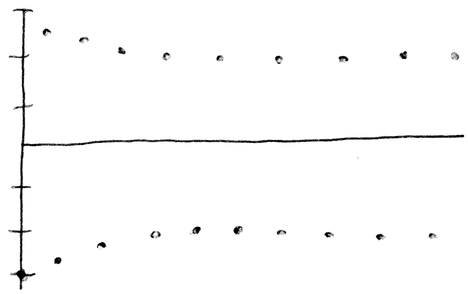
$3{,}010\, r^3 = 3{,}010{,}000 \rightarrow r^3 = 1{,}000 \rightarrow \boxed{r = 10}$

$a_1 = \frac{3{,}010}{10^3} = \frac{3{,}010}{1{,}000} = \boxed{3.010}$

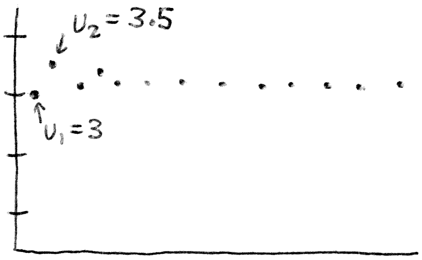
$\boxed{a_n = 3.010(10)^{n-1}}$

25. $a_n = (-1)^n \dfrac{2n+1}{n}$

Mode: Seq with Dot (not connected)



29. $U_1 = 3, \quad U_n = 5 - \frac{1}{2} U_{n-1}$



Seq, Dot

n Min = 2

$U_n = 5 - \frac{1}{2} \cdot U(n-1)$

$U(n\text{Min}) = 5 - \frac{1}{2}(3) = 5 - 1.5 = 3.5$

33. $\displaystyle \lim_{n \to \infty} \frac{2n^2 - n - 1}{5n^2 + n + 2} = \lim_{n \to \infty} \frac{2\cancel{n^2}}{5\cancel{n^2}} = \boxed{\frac{2}{5}}$ (converges)

37. $\displaystyle \lim_{n \to \infty} (1.1)^n = 1.1^{\infty} = \boxed{\infty}$ (diverges)

41. $-1 \le \sin n \le 1$ so $\dfrac{-1}{n} \le \dfrac{\sin n}{n} \le \dfrac{1}{n}$

$\displaystyle \lim_{n \to \infty} \frac{-1}{n} = \frac{-1}{\infty} = 0$ and $\displaystyle \lim_{n \to \infty} \frac{1}{n} = \frac{1}{\infty} = 0$

$\displaystyle \lim_{n \to \infty} \frac{-1}{n} \le \lim_{n \to \infty} \frac{\sin n}{n} \le \lim_{n \to \infty} \frac{1}{n}$, so $\displaystyle 0 \le \lim_{n \to \infty} \frac{\sin n}{n} \le 0$

$\displaystyle \lim_{n \to \infty} \frac{\sin n}{n}$ has to be 0 by the Sandwich Theorem.

$a_n = \dfrac{\sin n}{n}$ converges to 0.

45. $a_n = \dfrac{2n-1}{n}$

$\lim\limits_{n\to\infty} \dfrac{2n-1}{n} = \lim\limits_{n\to\infty} \dfrac{2\cancel{n}}{\cancel{n}} = 2 \to$ converges to 2, so $\boxed{B}$

46. $b_n = (-1)^n \dfrac{3n+1}{n+3}$

$\lim\limits_{n\to\infty} \dfrac{3n+1}{n+3} = \lim\limits_{n\to\infty} \dfrac{3\cancel{n}}{\cancel{n}} = 3$, but $(-1)^n$ alternates + & − signs,
so b_n diverges to 3 and −3, so $\boxed{C}$

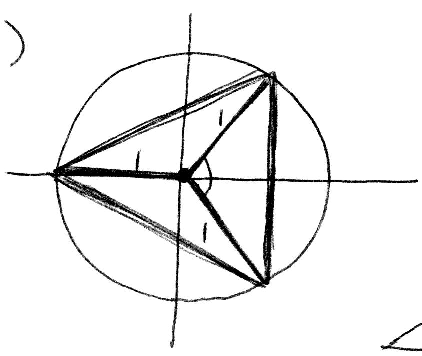
47. $c_n = \dfrac{n+1}{n}$

$\lim\limits_{n\to\infty} \dfrac{n+1}{n} = \lim\limits_{n\to\infty} \dfrac{n}{n} = 1$, so converges to 1, so $\boxed{D}$

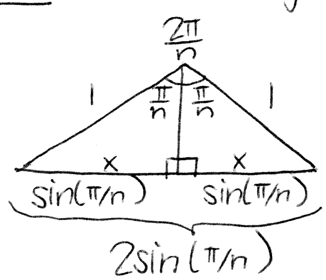
48. $d_n = \dfrac{4}{n+2}$

$\lim\limits_{n\to\infty} \dfrac{4}{n+2} = \lim\limits_{n\to\infty} \dfrac{4}{n} = \dfrac{4}{\infty} = 0 \to$ converges to 0, so $\boxed{A}$

55. a)



$n = 3$
3 isosceles triangles
Central angle $= \dfrac{2\pi}{n}$



$\sin\left(\frac{\pi}{n}\right) = \dfrac{O}{H} = \dfrac{x}{1}$

$x = 1 \cdot \sin\left(\frac{\pi}{n}\right) = \sin\left(\frac{\pi}{n}\right)$

Perimeter = # of sides · side length = $n \cdot 2\sin\left(\frac{\pi}{n}\right) = \boxed{2n\sin\left(\frac{\pi}{n}\right)}$

b) $\lim\limits_{n\to\infty} a_n = \lim\limits_{n\to\infty} 2n\sin\left(\frac{\pi}{n}\right) = 2\infty \cdot \sin\left(\frac{\pi}{\infty}\right) = \infty \cdot 0$ (indeterminate form)

Let $x = \dfrac{\pi}{n}$, so $n = \dfrac{\pi}{x}$. When $n\to\infty$, $x = \dfrac{\pi}{\infty} = 0$.

$\lim\limits_{n\to\infty} 2n\sin\left(\frac{\pi}{n}\right) = \lim\limits_{x\to 0} 2\frac{\pi}{x} \cdot \sin x = \lim\limits_{x\to 0} 2\pi \cdot \dfrac{\sin x}{x} = 2\pi \cdot 1 = \boxed{2\pi}$